The Performance of the Approximate Finite Difference Method of the Wavelength Division Multiplex Mesh Networks With Dedicated Optical Path Protection

Stefanos Mylonakis, *member*, *IEEE*

***Abstract*— The arithmetical methods solve successfully the survivability problems of the telecommunication networks. Such methods are the methods of the finite differences. These methods are either accurate ones or approximation ones. In this paper, the performance of three software models based on approximate polynomial finite difference method are studied and the results are compared with the corresponded of the accurate polynomial finite difference one. Our purpose is to assist to select the best model. The findings give the same results for two of three but the simpler is preferable and changing Turbo Pascal data types the approximate method is converted to accurate one.**

Index Terms— **error, finite differences, accurate polynomial method, approximate polynomial method, dedicated protection.**

I. Introduction

More and more infrastructures of the world are dependent on computer systems that have several requirements. Communications, transportation, medicine and so on are all increasingly involving processes and activities that require precise observance of several constraints. As a consequence, researchers have proposed and evaluated a plethora of such systems. The evaluation criterion of the performance of these three software models is the accuracy of the results. Optical WDM (Wavelength Division Multiplex) mesh network designs have been based on suitable mathematics. That is assumed that all of the values and results of mathematical computations are presented with high precision and reduced precision. In this paper, the performance of three software models based on approximate polynomial finite difference method is studied and results are compared with the corresponded of the accurate polynomial finite difference one. To obtain the best results several floating-point formats are used but the floating-point formats are simple methods and inadequate to delete errors. When these methods are not used the degradations can be severe [4] and [5]. The WDM optical mesh networks are high capacity telecommunication networks based on optical technologies allowing multiple frequency separated channels to co-transmit. For WDM system with many channels on a single fibre, a fibre cut would result in multiple failures, causing many independent systems to fail.The optical path with dedicated 1+1 protection on optical layer of optical WDM mesh networks can perform protection switching faster and more economically.

The optical path with dedicated 1+1 protection problem is solved using both polynomial methods of the finite differences ccalculating the final available capacity for each link as well as the total network available capacity by two polynomial finite difference methods, the accurate one and the approximate one. For the approximate one, three models are done, each of them with different calculation for the coefficient ($\alpha_{i,r}$) which are called first model, second model and third model. In the first one, the fractions in the coefficients have replaced by approximate values (table 2), in the second one, the fractions keep their values (table 3) and in the third one, the coefficient calculations are done changing the calculation algorithms (table 4). The results are calculated and compared for each data representation type of Turbo Pascal, each other and with the corresponded of the accurate finite difference method (table 5). This research has been done in relation to the methods and the problems associated with planning, protection and restoration of optical networks. In [1] the authors present Optical Transport Network (OTN) evolution from an operator's point of view, including the history of the transport network, the role of the OTN, and the motivations and requirements for OTN evolution. For WDM system with many channels on a single fiber, a fiber cut would results multiple failures, causing many independent systems to fail [2][3][6][7][8][11][12]. There are also several approaches to

Manuscript received November 14, 2014. St. T. Mylonakis is with the University of Athens, Athens, Attica, GREECE (corresponding author to provide phone: 00302108814002; fax: 00302108233405; e-mail: smylo@ otenet.gr).

ensure fiber network survivability [2][3][6][7][11]. Network survivability is defined as the capability of a communication network to resist any link or node interruption or disturbance of service, by any catastrophes. The existing methods in solving these problems use special algorithms. We suggest a proposal that it is an approach based on the finite differences polynomial methods and represents the detail algorithm description and its program. The advantage of this approach is that the polynomial methods of finite differences solve the same telecommunication problem (of this kind) by two different methods simultaneously and they can verify each other in accepted tolerances. These methods also can compare with the linear one [13] and it is another verification way.

The following analysis presents the solution of the problems associated with the survival optical networks on the basis of the finite differences and the corresponded problem is solved. The role of the Difference Calculus is in the study of the Numerical Methods. Computer solves these Numerical Methods. The subject of the Difference Equations [9] is in the treatment of discontinuous processes. The network final available capacity is revealed as a difference equation because the final available capacity of the individual working optical fibers is also a difference equation. The reduction of the available capacity of each working optical fiber is a discontinuous process when connection groups of several sizes pass through it. In [12], the authors begin with an overview of the existing strategies for providing transport network survivability and continue with an analysis of how the architectures for network survivability may evolve to satisfy the requirements of emerging networks. In [13], the author presents the finite differences, their methods and their problems when they are used to solve problems of this kind. In [14], the authors lay the foundation of switching node designs for future WDM-SDM optical networks. In [15], the authors present a simulation study of the Model Evaluation Criterion MMRE (Mean Magnitude of Relative Error). In [16], the authors provide an overview of the latest developments and possible approaches with respect to flexible optical networking and the emerging benefits that spatially flexible networking approaches can offer. In [17], the authors propose a control and management architecture to allow the network to be dynamically operated. In [18], the authors present a generic step-by-step methodology for evaluating the total cost of migrating from a capacity – exhausted WDM network to different upgraded alternatives.

This paper is broken down in the following sections: Section II shows how the finite differences are used for each optical fiber and illustrates the optical fiber final available capacity; Section III describes the problem, its formulation, its algorithm, an example and the proposals with discussion; Section IV draws conclusion and finally ends with the references.

II. THE OPTICAL FIBER AND THE FINITE DIFFERENCES

A. Finite differences and the optical WDM mesh networks

Before studying finite differences and their use in optical WDM mesh networks survivability, it is necessary to provide a short comprehensive presentation of the finite differences computation. Let's assume that $y_1,y_2,\ldots,y_n$ is a sequence of numbers in which the order is determined by the index n. The number n is an integer and the y_n can be regarded as a function of n, an independent variable with function domain the natural numbers and it is discontinuous. Such a sequence shows the available capacity reduction of a telecommunication fibre network link between two nodes when the telecommunication traffic of 1,2, …, n source-destination node pairs pass through. It is assumed that the telecommunication traffic unit is the optical channel that is one wavelength (1λ). The telecommunications traffic includes optical connections with their protections. The total connections of a node pair form its connection group. The first order finite differences represent symbolically the connection group of each node pair that passes through a fiber. This connection group occupies the corresponding number of optical channels and it is the bandwidth that is consumed by connections of a node pair through this fiber. The first order finite differences are used to represent the connection groups in optical channels of the node pairs that pass through an optical fibre. An equation of the first order finite differences gives the available capacity of an optical fiber network link when a connection group passes through it. This available capacity is provided for the connection groups of the other node pairs that their connections will pass through this optical fiber. When the first connection group of Δy_1 connections passes through an optical fiber network link with installed capacity of y_1 optical channels the first order finite difference equation gives the available capacity $y_{2\,(y_1+1)}$ which is written as following

$$y_{1+1} = y_1 - \Delta y_1$$

The sequence Δy_1, Δy_2, $\Delta y_3,\ldots,\Delta y_n$ represents the connection groups that pass through this optical fiber network link. When Δy_1 subtracted from y_1, creates y_2, when Δy_2 subtracted from y_2, creates $y_3,\ldots,$ when Δy_n subtracted from y_n creates y_{n+1} which is the total unused available capacity of this optical fiber. It means the followings.

$$
\begin{aligned}
y_{0+1} &= y_0\\
y_{1+1} &= y_1 - \Delta y_1\\
y_{2+1} &= y_2 - \Delta y_2\\
y_{3+1} &= y_3 - \Delta y_3\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
y_{(n-1)+1} &= y_{(n-1)} - \Delta y_{(n-1)}\\
y_{n+1} &= y_n - \Delta y_n
\end{aligned}
$$

Thus the total unused available capacity of each network optical fiber is calculated after n connections groups pass through it.

B. Polynomial Functions

The total unused available capacity of each network optical fiber is also written as a polynomial function, and there are two polynomial function methods. The assessment of the polynomial function coefficients is done with the values that the polynomial function represents for 1, 2,…, n, n +1. The values of the function y_{n+1} for each n must be integral because each value represents optical channels. There are more details in the [11] but for helping the reader we write them again.

The general form of a polynomial function that gives the available capacity of the optical fiber network link after the serving n connection groups, is as follows

$$y_{n+1} = \sum_{r=0}^{n} \alpha_r * (n+1)^r \qquad (1)$$

The assessment of the polynomial function coefficients is done with the values that the polynomial function represents for 1,2,…, *n*, *n* +1 .The values of the function y_{n+1} for each *n* must be integral because each value represents optical channels.

- If the equation (1) is written analytically as follows

$y_{0+1}= \alpha_0*(0+1)^0$

$y_{1+1}= \alpha_0*(1+1)^0 +\alpha_1*(1+1)^1$

$y_{2+1}= \alpha_0*(2+1)^0 +\alpha_1*(2+1)^1+\alpha_2*(2+1)^2$

$y_{3+1}= \alpha_0*(3+1)^0 +\alpha_1*(3+1)^1+\alpha_2*(3+1)^2 +\alpha_3*(3+1)^3$

………………………………………………………….

$y_{n+1}= \alpha_0*(n+1)^0 +\alpha_1*(n+1)^1+\alpha_2*(n+1)^2 +\ldots+\alpha_n*(n+1)^n$

The value of the function has high accuracy of 15 decimal digits. This method is an *accurate* one.

-If the equation (1) is written analytically as follows

$y_{0+1}= \alpha_0*(0+1)^0 +\alpha_1*(0+1)^1+\alpha_2*(0+1)^2 +\ldots+\alpha_n*(0+1)^n$

$y_{1+1}= \alpha_0*(1+1)^0 +\alpha_1*(1+1)^1+\alpha_2*(1+1)^2 +\ldots+\alpha_n*(1+1)^n$

$y_{2+1}= \alpha_0*(2+1)^0 +\alpha_1*(2+1)^1+\alpha_2*(2+1)^2 +\ldots+\alpha_n*(2+1)^n$

$y_{3+1}= \alpha_0*(3+1)^0 +\alpha_1*(3+1)^1+\alpha_3*(3+1)^2 +\ldots+\alpha_n*(3+1)^n$

………………………………………………………….

$Y_{n+1}= \alpha_0*(n+1)^0 +\alpha_1*(n+1)^1+\alpha_2*(n+1)^2 +\ldots+\alpha_n*(n+1)^n$

The value of the function has reduced accuracy and depends of the value of *n*. This method is *an approximated* one.

The above polynomial functions are systems of (n+1) equations with (n+1) unknown coefficients. The values of the coefficients depend of the number of the connection groups and the connections of each connection group. The 1st method is more accurate than 2nd one because only one factor is added to new equation when the polynomial degree increases versus one factor is added for all equations respectively. The equation, (1) must be greater or equal to zero, for full servicing all connection groups that pass through an optical fiber. So the number of connections on each link is bounded.

C. *The number, the order and the size of $\Delta y_{i,j}$ are critical*

The number, the order and the size of $\Delta y_{i,j}$ are critical. It is showed and proved below. The symbol $\Delta y_{i,j}$ is explained in the table with the symbols of this paper. The polynomials that calculate the available capacity of each optical link for each polynomial method for all possible values up to four and for the following cases are represented. The WDM system capacity is 30 λ.

For the accurate method it is written.

-If no one-connection group passes through an optical link the polynomial function is constant, etc.

$\underline{y_{j,0+1}= \alpha_0*(0+1)^0}$

$y_{j,0+1}=30.$

-If only one-connection group passes through an optical link the polynomial function is of the first degree.

$\underline{\Delta y_{i,1}, \quad y_{i,1+1}= \alpha_0*(1+1)^0 +\alpha_1*(1+1)^1}$

1 $\quad y_{i,1+1}= 30*(1+1)^0 -(1/2)*(1+1)^1 =29$

2 $\quad y_{i,1+1}= 30*(1+1)^0 -(2/2)*(1+1)^1=28$

3 $\quad y_{i,1+1}= 30*(1+1)^0 -(3/2)*(1+1)^1=27$

4 $\quad y_{i,1+1}= 30*(1+1)^0 -(4/2)*(1+1)^1=26$

-If only two-connection groups pass through an optical link the polynomial function is of the second degree.

$\underline{\Delta y_{i,1}, \Delta y_{i,2}, y_{i,1+1}= \alpha_0*(2+1)^0 +\alpha_1*(2+1)^1+\alpha_2*(2+1)^2}$

1 , 1, $y_{i,2+1}= 30*(2+1)^0 -0.5*(2+1)^1- 5.55555555555429E-2 *(2+1)^2 =28$

2 , 1, $y_{i,2+1}= 30*(2+1)^0 -1.0*(2+1)^1-0.00000000000000E+0*(2+1)^2 =27$

1 , 2, $y_{i,2+1}= 30*(2+1)^0 -0.5*(2+1)^1-1.66666666667420E-1*(2+1)^2 =27$

3 , 1, $y_{i,2+1}= 30*(2+1)^0 -1.5*(2+1)^1+5.55555555555429E-2 *(2+1)^2=26$

2 , 2, $y_{i,2+1}= 30*(2+1)^0 -1.0*(2+1)^1-1.11111111111086E-1*(2+1)^2 =26$

1 , 3, $y_{i,2+1}= 30*(2+1)^0 -0.5*(2+1)^1-2.77777777777828E-1*(2+1)^2 =26$

4 , 1, $y_{i,2+1}= 30*(2+1)^0 -2.0*(2+1)^1+1.11111111111086E-1*(2+1)^2=25$

3 , 2, $y_{i,2+1}= 30*(2+1)^0 -1.5*(2+1)^1-5.55555555555429E-2 *(2+1)^2 =25$

2 , 3, $y_{i,2+1}= 30*(2+1)^0 -1.0*(2+1)^1-2.22222222221720E-1*(2+1)^2=25$

1 , 4, $y_{i,2+1}= 30*(2+1)^0 -0.5*(2+1)^1-3.88888888886870E-1*(2+1)^2=25$

4 , 2, $y_{i,2+1}= 30*(2+1)^0 -2.0*(2+1)^1+0.00000000000000E+0*(2+1)^2=24$

3 , 3, $y_{i,2+1}= 30*(2+1)^0 -1.5*(2+1)^1-1.66666666667420E-1*(2+1)^2 =24$

2 , 4, $y_{i,2+1}= 30*(2+1)^0 -1.0*(2+1)^1-3.33333333333485E-1*(2+1)^2=24$

e.t.c

This method is an accurate one but if all coefficients are not written completely with 15 digits there are errors in the results. The above ones are rounded to the closest integer to agree with the real ones. The same are valid for the follows.

For the approximate method it is written.

-If no one-connection group passes through an optical link the polynomial function is constant.

$\underline{y_{j,0+1}= \alpha_0*(0+1)^0}$

$y_{j,0+1}=30.$

-If only one-connection group passes through an optical link the polynomial function is of the first degree.

$\underline{\Delta y_{i,1}, \quad y_{i,1+1}= \alpha_0*(1+1)^0 +\alpha_1*(1+1)^1}$

1 $\quad y_{i,1+1}= (30+1)*(1+1)^0 -1*(1+1)^1 =29$

2 $\quad y_{i,1+1}= (30+2)*(1+1)^0 -2*(1+1)^1=28$

3 $\quad y_{i,1+1}= (30+3)*(1+1)^0 -3*(1+1)^1=27$

4 $\quad y_{i,1+1}= (30+4)*(1+1)^0 -4*(1+1)^1=26$

-If only two-connection groups passes through an optical link the polynomial function is of the second degree.

$\underline{\Delta y_{i,1}, \Delta y_{i,2}, y_{i,1+1}= \alpha_0*(2+1)^0 +\alpha_1*(2+1)^1+\alpha_2*(2+1)^2}$

1 , 1, $y_{i,2+1}= 31*(2+1)^0 -1.0*(2+1)^1+0.000*(2+1)^2 =28$

2 , 1, $y_{i,2+1}= 33*(2+1)^0 -3.5*(2+1)^1+0.500*(2+1)^2=27$

1 , 2, $y_{i,2+1}= 30*(2+1)^0 +0.5*(2+1)^1-0.500*(2+1)^2 =27$

3 , 1, $y_{i,2+1}= 35*(2+1)^0 -6.0*(2+1)^1+1.000*(2+1)^2 =26$

2 , 2, $y_{i,2+1}= 32*(2+1)^0 -2.0*(2+1)^1-0.000*(2+1)^2=26$

1 , 3, $y_{i,2+1}= 29*(2+1)^0 +2.0*(2+1)^1-1.000*(2+1)^2=26$

4 , 1, $y_{i,2+1}= 37*(2+1)^0 -8.5*(2+1)^1+1.500*(2+1)^2=25$

3 , 2, $y_{i,2+1}= 34*(2+1)^0 -4.5*(2+1)^1+0.500*(2+1)^2 =25$

2 , 3, $y_{i,2+1}= 31*(2+1)^0 -0.5*(2+1)^1-0.500*(2+1)^2=25$

1 , 4, $y_{i,2+1}= 28*(2+1)^0 +3.5*(2+1)^1-1.500*(2+1)^2=25$

4 , 2, $y_{i,2+1}= 36*(2+1)^0 -7.0*(2+1)^1+1.000*(2+1)^2=24$

3 , 3, $y_{i,2+1}= 33*(2+1)^0 -3.0*(2+1)^1+0.000*(2+1)^2 =24$

2 , 4, $y_{i,2+1}= 30*(2+1)^0 +1.0*(2+1)^1-1.000*(2+1)^2=24$

e.t.c

III. THE PROBLEM AND ITS SOLUTION

A. *The problem*

The network topology and other parameters are known as WDM and optical fiber capacity, one optical fiber per link with an extension to a 1+1 fiber protection system. So this network is characterized by one working fiber per link, edges of two links, links of two optical fibers, one for working and one for protection. The connections are lightpaths originating in the source nodes and terminating at the destination nodes proceeding from preplanned optical working paths. Additionally, the same number of optical paths is preselected for the preplanned fully disjoint backup paths, (1+1 dedicated protection connection). So the connections are protected. The connections of the same node pair form a group along the network. The preplanned protection paths do the dedicated protection of the connection groups. So a suitable number of wavelengths per link along the network is used. The solution is the calculation of the final available capacity of the network for a given table. This table contains the number of the node pairs, the node pairs and the number of the connections of each node pair when their working and protection paths are preplanned. For the approximate one, three models are done, each of them with different calculation for the coefficient ($\alpha_{i,r}$) which are called first model, second model and third model. In the first one, the fractions in the coefficients have replaced by approximate values (table 2), in the second one, the fractions keep their values (table 3) and in the third one, the coefficient calculations are done changing the calculation algorithms (table 4). The results are calculated and compared for each data representation type of Turbo Pascal, each other and with the corresponded of the accurate finite difference method (table 5). In [13] I have proved that the linear finite difference method (which is an ILP method with the disadvantage the large matrix A (2pxn) which occupies large memory and has difficult treatment)) and the polynomial accurate method is not introducing errors in the solution of the problem. The polynomial approximate one does it but these errors could be minimized with several ways and the results are improved. So all requests for connection are satisfied and form connections

TABLE 1
THE SYMBOLS OF THIS PAPER

S/N	Symbol	Comments
1	q	Node number
2	p	Edge number
3	G(V,E)	Network graph
4	V(G)	Network node set
5	E(G)	Network edge set
6	2p	The number of working and backup fiber for 1+1 line protection
7	n	The number of source – destination nodes pairs of the network
8	n(i)	The total number of the connection groups that passes through the fibre (*i*) and means that each fibre has different number of connection groups pass through it
9	$n(i)_w$	The number of the working connection groups that passes through the fibre (*i*) and means that each fibre has different number of connection groups pass through it
10	$n(i)_p$	The number of the protection connection groups that passes through the fibre (*i*) and means that each fibre has different number of connection groups pass through it
11	K	The number of the wavelengths channels on each fiber that is the WDM system capacity
12	Cinst	The total installed capacity
13	Cav	The total available capacity
14	Cw	The total working capacity
15	Cpr	The total protection capacity
16	C_b	The total busy capacity
17	$\Delta y_{i,j}$	The first order of finite difference that corresponds to a group of optical connections that passes through the optical fiber ***i*** with serial number ***(j)*** respectively.

TABLE 2
THE COEFFICIENT CALCULATIONS FOR THE FIRST MODEL OF APPROXIMATE METHOD

First Model
n=1,two coefficients
$\alpha_0 = 2Y_{0+1} - Y_{1+1}$
$\alpha_1 = Y_{1+1} - Y_{0+1}$
n=2,three coefficients
$\alpha_2 = (1/2)Y_{2+1} - Y_{1+1} + (1/2)Y_{0+1}$
$\alpha_1 = (-3/2)Y_{2+1} + 4Y_{1+1} - (5/2)Y_{0+1})$
$\alpha_o = Y_{2+1} - 3Y_{1+1} + 3Y_{0+1}$
n=3,four coefficients
$\alpha_3 = 0.1667Y_{3+1} - 0.5Y_{2+1} + 0.5Y_{1+1} - 0.1667Y_{0+1}$
$\alpha_2 = - Y_{3+1} + 3.5Y_{2+1} - 4Y_{1+1} + 1.5Y_{0+1}$
$\alpha_1 = 1.8333Y_{3+1} - 7Y_{2+1} + 9.5Y_{1+1} - 4.3333Y_{0+1}$
$\alpha_o = -Y_{3+1} + 4Y_{2+1} - 6Y_{1+1} + 4Y_{0+1}$
n=4,five coefficients
$\alpha_4 = -0.0417Y_{4+1} - 0.1667Y_{3+1} + 0.25Y_{2+1} - 0.1667Y_{1+1} + 0.0417 Y_{0+1}$
$\alpha_3 = -0.4167Y_{4+1} + 1.8333Y_{3+1} - 3Y_{2+1} + 2.1667Y_{1+1} - 0.5833Y_{0+1}$
$\alpha_2 = 1.4583Y_{4+1} - 6.8333Y_{3+1} + 12.25Y_{2+1} - 9.8333Y_{1+1} + 2.9583 Y_{0+1}$
$\alpha_1 = -2.0833Y_{4+1} + 10.1667Y_{3+1} - 19.5Y_{2+1} + 17.8333Y_{1+1} - 6.4167 Y_{0+1}$
$\alpha_o = Y_{4+1} - 5Y_{3+1} + 10Y_{2+1} - 10Y_{1+1} + 5Y_{0+1}$

TABLE 3
THE COEFFICIENT CALCULATIONS FOR THE SECOND MODEL OF APPROXIMATE METHOD

Second Model
n=1,two coefficients
$\alpha_0 = 2Y_{0+1} - Y_{1+1}$
$\alpha_1 = Y_{1+1} - Y_{0+1}$
n=2,three coefficients
$\alpha_2 = (1/2)Y_{2+1} - Y_{1+1} + (1/2)Y_{0+1}$
$\alpha_1 = (-3/2)Y_{2+1} + 4Y_{1+1} - (5/2)Y_{0+1})$
$\alpha_o = Y_{2+1} - 3Y_{1+1} + 3Y_{0+1}$
n=3,four coefficients
$\alpha_3 = (1/6)(Y_{3+1} - 3Y_{2+1} + 3Y_{1+1} - Y_{0+1})$
$\alpha_2 = (1/2)(- 2Y_{3+1} + 7Y_{2+1} - 8Y_{1+1} + 3Y_{0+1})$
$\alpha_1 = (1/12)(22Y_{3+1} - 84Y_{2+1} + 114Y_{1+1} - 52Y_{0+1})$
$\alpha_o = -Y_{3+1} + 4Y_{2+1} - 6Y_{1+1} + 4Y_{0+1}$
n=4,five coefficients
$\alpha_4 = (1/24)(Y_{4+1} - 4Y_{3+1} + 6Y_{2+1} - 4Y_{1+1} + Y_{0+1})$
$\alpha_3 = (1/6)(-2.5Y_{4+1} + 11Y_{3+1} - 18Y_{2+1} + 13Y_{1+1} - 3.5Y_{0+1})$
$\alpha_2 = (1/24)(35Y_{4+1} - 164Y_{3+1} + 294Y_{2+1} - 236Y_{1+1} + 71Y_{0+1})$
$\alpha_1 = (1/48)(-100Y_{4+1} + 488Y_{3+1} - 936Y_{2+1} + 856Y_{1+1} - 308Y_{0+1})$
$\alpha_o = Y_{4+1} - 5Y_{3+1} + 10Y_{2+1} - 10Y_{1+1} + 5Y_{0+1}$

B. Formulation

The network is assumed to be an optical mesh network with the circuit switched (or packet switched but the packets are adjusted to follow preplanned paths) as a graph. Each vertex represents the central telecommunications office (CO) with the OXC while each edge represents two links. Each edge link has a couple of optical fibers. All optical fibers have the same capacity as the WDM system. All nodes are identical. The numbers of working and protection connections that pass through each optical fiber are different. Finite difference polynomial equations are calculated for each optical fiber, for polynomial accurate method and for the polynomial approximate ones. For all network links, the general equation of the polynomial function has two column matrices, the left one that is equals with the right one. When all connections have been set up then each element of the column matrix must be greater or equal to zero. In other cases some connections are not possible. The total final available capacity of the network for the polynomial methods is given by the equation (2). This is the formulation of the *polynomial function* method problem.

$$\sum_{i=1}^{2p} y_{i,n(i)+1} = \sum_{i=1}^{2p} \sum_{r=0}^{n(i)} \alpha_{i,r} * (n(i)+1)^r$$

$$y_{i,n(i)+1} >= 0 \quad ,n(i)>0$$

TABLE 4
THE COEFFICIENT CALCULATIONS FOR THE THIRD MODEL OF APPROXIMATE METHOD

Third Model
n=1,two coefficients
$\alpha_0 = 2Y_{0+1} - Y_{1+1}$
$\alpha_1 = Y_{1+1} - Y_{0+1}$
n=2,three coefficients
$\alpha_2 = (1/2)(Y_{2+1} - 2Y_{1+1} + Y_{0+1})$
$\alpha_1 = Y_{2+1} - Y_{1+1} - 5a_2$
$\alpha_o = Y_{0+1} - a_1 - a_2$
n=3,four coefficients
$\alpha_3 = (1/6)(Y_{3+1} - 3Y_{2+1} + 3Y_{1+1} - Y_{0+1})$
$\alpha_2 = (1/2)(Y_{3+1} - 2Y_{2+1} + Y_{1+1} - 18a_3)$
$\alpha_1 = Y_{3+1} - Y_{2+1} - 7a_2 - 37a_3$
$\alpha_o = Y_{0+1} - a_1 - a_2 - a_3$
n=4,five coefficients
$\alpha_4 = (1/24)(Y_{4+1} - 4Y_{3+1} + 6Y_{2+1} - 4Y_{1+1} + Y_{0+1})$
$\alpha_3 = (1/6)(Y_{3+1} - 3Y_{2+1} + 3Y_{1+1} - Y_{0+1} -60a_4)$
$\alpha_2 = (1/2)(Y_{2+1} - 2Y_{1+1} + Y_{0+1} - 12a_3 - 50a_4)$
$\alpha_1 = Y_{1+1} - Y_{0+1} - 3a_2 - 7a_3 - 15a_4$
$\alpha_o = Y_{1+1} - 2a_1 - 4a_2 - 8a_3 - 16a_4$

For n=5 and n=6, the six and seven coefficients are calculated with the same way for the models second and third ones. The values of the coefficients ($\alpha_{i,r}$) or simpler (α_r) depend of the number of the connection groups and the connections of each connection group. The coefficients ($\alpha_{i,r}$) for each link are calculated by a linear system of (n+1) equations with (n+1) unknown values. The calculated values of the coefficients give available capacity for the link which is a little different than the real one. This little different is ought to the errors of the coefficients. In this paper, these little differences could be calculated subtracting the available capacity of the approximate methods from the accurate one. To correct this problem several methods are used and showed in the tables 3,4 and 5 as well as in the example. The coefficients of table 2 introduce rounding and truncation errors. The coefficients of tables 3 and 4 have reduced errors.The rounding errors are rise by approximating a fraction with periodic decimal expansion by a finite decimal value or truncate it to the smaller value. In the accurate method, the corresponded coefficient calculations are simpler, table 5 and introduce smaller errors than approximate one.

TABLE 5
THE COEFFICIENT CALCULATIONS FOR THE MODEL OF ACCURATE METHOD

Accurate Model
n=1,two coefficients
$\alpha_0 = Y_{0+1}$
$\alpha_1 = (Y_{1+1} - \alpha_0)/2$
n=2,three coefficients
$\alpha_2 = (Y_{2+1} - \alpha_0 - \alpha_1 3)/9$
$\alpha_1 = (Y_{1+1} - \alpha_0)/2$
$\alpha_0 = Y_{0+1}$
n=3,four coefficients
$\alpha_3 = (Y_{3+1} - \alpha_0 - \alpha_1 4 - \alpha_2 16)/64$
$\alpha_2 = (Y_{2+1} - \alpha_0 - \alpha_1 3)/9$
$\alpha_1 = (Y_{1+1} - \alpha_0)/2$
$\alpha_0 = Y_{0+1}$
n=4,five coefficients
$\alpha_4 = (Y_{4+1} - \alpha_0 - \alpha_1 5 - \alpha_2 25 - \alpha_3 125)/625$
$\alpha_3 = (Y_{3+1} - \alpha_0 - \alpha_1 4 - \alpha_2 16)/64$
$\alpha_2 = (Y_{2+1} - \alpha_0 - \alpha_1 3)/9$
$\alpha_1 = (Y_{1+1} - \alpha_0)/2$
$\alpha_0 = Y_{0+1}$

C. The algorithm

Our algorithm describes the operation of the WDM optical fiber mesh network. TURBO PASCAL is used to program the models [4]. The algorithm has the following steps and phases.

First step *Network parameters*

Initially the following data are known: network topology, node number, edge number, link number per edge, working optical fiber number per link, protection optical fiber number per link, wavelength number per optical fiber, optical fiber numbering. This information allows the computer to draw a graph and an OXC is on the vertex of the graph [10]. Each edge corresponds to two links with opposite direction to each other. All fibers have the same wavelength number and all links the same fiber number. The computer reads the adjacency matrix and is informed about the network topology.

Second step *Connection selections*

In this step, the number of the connection node pair, the connection node pair selection for connections and the desired connection group size are done. The preplanned working and the protection optical paths for connections of every node pair are also provided.

Failure-free Network Phase

Third step *Wavelength allocation*

In this step, wavelength allocation is initiated. A working connection starts from the source node and progresses through the network occupying a wavelength on each optical fiber and switch to another fiber on the same or other wavelength by OXC, according to its preplanned working optical path up to arrive at the destination node. Simultaneously, the protection connection starts from the source node and progresses through the network occupying a wavelength on each optical fiber and switch to another fiber on the same or other wavelength by OXC, according to its preplanned protection optical path up to arrive at the destination node. So there is full and dedicated protection for this connection. The number of connections of each node pair is equal to its connection group size. After a connection (working as well as protection) has been established, the available capacity is also calculated. These results are for one connection.

Forth step. *Presentation of the finite differences*

The total available capacity of each optical fiber is calculated and represented. These results are for all connections.

Fifth step *Results and comparisons thereof*

Knowing the desired connection group size the total results are computed that are the total sum of the individual

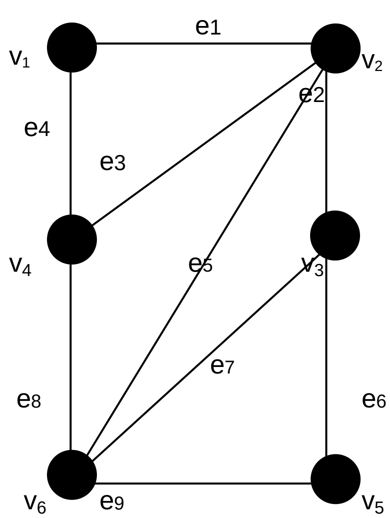

Fig. 1.The mesh topology of the network.

connection group size, the total installed capacity, the total protection capacity, the total busy capacity and the available capacity. These results are compared and showed in tables side by side for each model so that the differences to be looked.

Network with failure Phase

When a failure occurs and a link is cut, the optical fibers of this link are also cut and the optical fiber protection 1+1 and the network topology change. The connection groups that passed through the cut link are also cut and the restoration is carried out passing through the preplanned protection paths of other links. The computer is informed of the cut link and modifies suitably the network parameters. The cut optical fiber sets its wavelengths to zero. The connection groups that passing through the cut link set their using wavelengths to zero and through the others to free.

D. Example

The network and the results are presented shortly because the paper must be short. The topology of the network is presented by the graph G (V, E). The vertex set has q=6 elements and the edge set has p=9 elements. Each edge is an optical link of two directions with one working fiber for each direction. Thus there are 2*p=2*9=18 optical fibers. Connection groups transverse the mesh network and correspond to *n* source-destination node pairs. WDM system capacity has 30 wavelengths.

The problem is solved for *n=12 of 30 possible* connection groups. These have their order and sizes for each source-destination node pair, their working paths and their protection paths as shown in *table 6*. The results with finite differences are showed. It is obvious that the dedicated path protection mechanisms use more than 100% redundant capacity because their lengths are longer than their working paths. The total length of working paths is twenty, (20) and the total length of protection paths is twenty-eight, (28). Similarly for the same connections requested group size the capacity that is used by the protection paths is larger than the corresponding working paths.

TABLE 6
ORDER, SIZE, WORKING PATH, PROTECTION PATH OF EACH NODE PAIR

Node Pair [Si,Di]	**Node pair [v_i, v_j]**	**Working Path**	**Protection Path**	**Group size**
[S_1, D_1]	[v_1, v_2]	v_1, v_2	v_1, v_4, v_2	1
[S_2, D_2]	[v_1, v_3]	v_1, v_2, v_3	v_1, v_4, v_6, v_3	2
[S_3, D_3]	[v_1, v_5]	v_1, v_2, v_3, v_5	v_1, v_4, v_6, v_5	5
[S_4, D_4]	[v_2, v_3]	v_2, v_3	v_2, v_6, v_3	2
[S_5, D_5]	[v_2, v_4]	v_2, v_4	v_2, v_1, v_4	2
[S_6, D_6]	[v_2, v_5]	v_2, v_3, v_5	v_2, v_6, v_5	3
[S_7, D_7]	[v_3, v_4]	v_3, v_2, v_4	v_3, v_6, v_4	1
[S_8, D_8]	[v_3, v_6]	v_3, v_6	v_3, v_5, v_6	4
[S_9, D_9]	[v_4, v_1]	v_4, v_1	v_4, v_2, v_1	1
[S_{10}, D_{10}]	[v_4, v_5]	v_4, v_6, v_5	v_4, v_2, v_3, v_5	2
[S_{11}, D_{11}]	[v_5, v_4]	v_5, v_6, v_4	v_5, v_3, v_2, v_4	5
[S_{12}, D_{12}]	[v_6, v_1]	v_6, v_4, v_1	v_6, v_2, v_1	2

The synoptic presentation is used for the finite difference tables. So the numbers of connection groups that pass through each optical fiber are showed in the table 7. (Fiber, i) shows the optical fibers. The n(i) shows the number of the connection groups that pass through each optical fiber. The polynomials that calculate the total network available capacity for each method for the following cases are not represented. The matrix with dimension is (4x1) of the total network available residual capacity for each case is given by the table 8 for each type data representation of TURBO PASCAL. Column (1) gives the symbolism of each data representation of Turbo Pascal, column (2) gives the total network available capacity of the accurate method, column (3) gives the total network available capacity of the approximate method first model, given by table (2), column (4) gives the total network available capacity of the approximate method second model given by table (3) and column (5) gives the total network available capacity of the approximate method third model given by table (5).

TABLE 7
THE NUMBER OF CONNECTION GROUPS THAT PASS THROUGH EACH OPTICAL FIBER

Fiber,i	1	2	3	4	5	6
n(i)	3	3	5	2	3	3
Fiber,i	7	8	9	10	11	12
n(i)	4	2	2	1	4	1
Fiber,i	13	14	15	16	17	18
n(i)	2	2	3	3	2	3

Time complexity of that algorithm is 'order q^2, $O(s*q^2)$. The matrix of coefficients of the accurate method is low triangular and of the approximate one is square. So the approximate method needs more memory for solving. In the figures 2 and 3, the error of the total available capacity versus the WDM system capacity is showed for the cases of table 2, 3 and 4. The results for the several data types of Turbo Pascal for the approximate method show that data types changing could improve partially or fully the performance of the approximate method and make it more accurate when suitable care have be taken and there are not other error sources.

TABLE 8
THE TOTAL NETWORK AVAILABLE CAPACITY PER TYPE DATA REPRESENTATION

(1)	**(2)**	**(3)**	**(4)**	**(5)**
Single(4)	411	411,339935302734	410,999969482422	410,999969482422
Real(6)	411	411,339999999851	410,999999999534	410,999999999534
Double(8)	411	411,340000000000	411,000000000000	411,000000000000
Extended (10)	411	411,340000000000	411,000000000000	411,000000000000

E. Discussion and Proposals

Since the 1950s, various computer simulation techniques have become increasingly important research tools across a wide range of sciences. Software packages based on these techniques are also widely used in more applied fields, often in connection with computerized databases and electronic gathering devices. There are several advantages of simulation compared with using real data [15]. In this study, I use simulation to investigate a WDM mesh network with dedicated optical path protection comparing the simulation results of four models, three of approximate finite difference methods and one of accurate finite difference method. In this protection scheme, when a single failure of a cut link occurs or other failure on main optical path so that the main connection cuts then the connection is routed by backup path. The use of the polynomial methods of finite differences is possible for the study of the problems related to the protection and restoration of connections and has the advantage versus the linear method that is not using the large matrix with the active links of the network. The active links are the links that pass through the optical paths. For better presentation of this research a short example is used that depicts the results in these methods. The algorithm provides for each source-destination node pair and a desired connection group size, a value of the total available capacity of the network. The connection length depends on the number of hops. The network has a complete protection for optical path connection. It is a switch circuit network (or a packet switch network but the packets are adjusted to pass through preplanned paths) so that one light path corresponds to one optical connection. Different wavelengths may be used for each connection in each hop, so that wavelength conversion is used at each node. These methods solve problems with small networks because when the connections groups that pass through a link increases, then the number of the (n+1) equations with n+1 unknown of the linear system also increases and it is difficult to be solved to calculate the coefficients. The accurate method is easier and more accurate than the approximate one because only one factor is added to each new equation when the polynomial degree increases versus one factor is added for all equations. In Turbo Pascal for the PC, the best precision is 15 decimal digits and some differences that appeared are ought to the errors of the polynomial methods. For the precision, the *different* types are used that are floating point formats and provide good dynamic range in addition to high precision. The model of table 2 produce more errors and losses in accuracy (loss significant digits), so that anyone type can not delete them because they are simple approaches changing the variable declarations without no other modifications to the program. The other models of table 3 and 4, overcome these problems using more exact numbers and changing the algorithm, revising the equations of the coefficients. The type single gives a lot of problems, after the type real reduces the problems in the higher degree polynomials, the type double deletes the problems as well as the type extended. The last type is used when the type double can not delete the problems. For small networks, Turbo Pacal data types can convert the approximate method to an accurate one when there are any types of errors. For the table 7, when none group goes through the optical fiber, then the degree of the polynomial function is 0, when one group goes through, then the degree of the polynomial function is 1, when two groups go through, then the degree of the polynomial function is 2, etc.

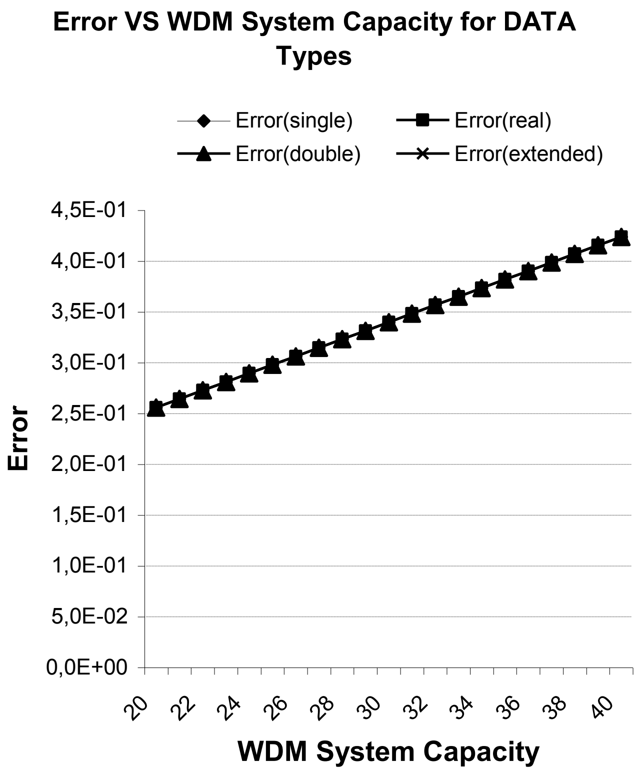

Fig 2.The error of the total available capacity versus the WDM system capacity for the case of table 2

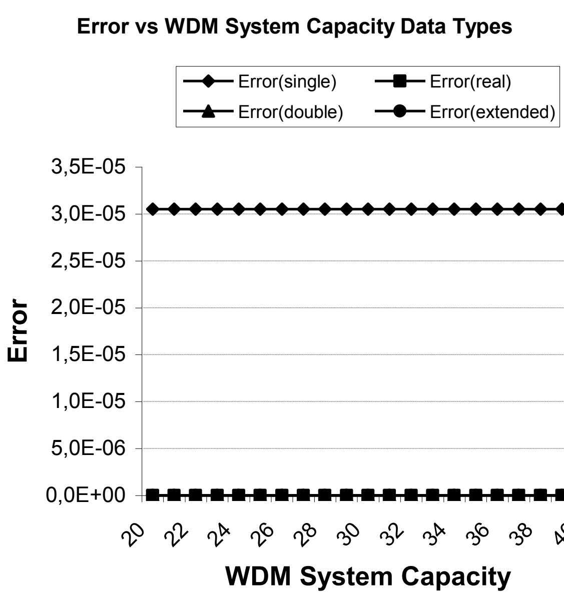

Fig 3.The error of the total available capacity versus the WDM system capacity for the cases of table 3 and 4

IV. CONCLUSION

In this paper, the performance of three models of the approximate polynomial method of finite differences are studied and compared for the dedicated protection optical path method of the WDM mesh networks. The approximate polynomial method of finite differences makes it possible to research and study the dedicated protection problems of the optical paths but more attention must pay. So a more suitable, accurate arithmetical method is produced to solve telecommunication problems such as planning of a completely protected network, complete protection for any failure occurs on optical path, node or link e. t. c. The approximate method could be converted to an accurate one using the several data types of Turbo Pascal and there are not any types of errors. This conversion is full for small networks but may be partial for larger networks.

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Stefanos Mylonakis (M,96) member of IEEE since 1996, was born in Chanea, Greece in 1954. He obtained a Bachelors degree in Physics from the University of Patras and two Master's degrees, in Radioelectrology (1984) and Electronic Automation (1987), both from the University of Athens. He was working as expert telecommunications engineer, engineering management in OTE (Hellenic Telecommunications Organisation).