Modeling and Control of a Two Wheeled Machine: A Genetic Algorithm-Based Optimization Approach

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***Abstract*-This work carries out design and implementation of a PID control algorithm utilizing genetic algorithm as an optimization technique for a novel design of a two-wheeled vehicle with an extended rod. The vehicle design offers an additional feature to the conventional inverted pendulum on two wheels. The intermediate body (IB) of the vehicle is composed of two co-axial parts connected by a linear actuator and with a payload attached to the end of the upper part. The linear actuator allows the payload to move up and down along the IB of the vehicle. Considering the various positions, speeds and different sizes of a payload, carried by the vehicle, while maintaining the entire vehicle balanced is the main anxiety of the current study. Dynamic modeling of the system is based on utilizing Lagrangian formulation to drive the system equations of motion.**

Keywords: Genetic algorithgm, PID control, two wheeled vehicle, Lagrangian dynamic formulation.

I. INTRODUCTION

The nature of a two-wheeled vehicle poses several interesting control questions. For instance, while a person occupies the vehicle, their mass changes the centre of gravity of the vehicle which in turn has an impact on the control technique used. Industrial applications of such vehicles will arise to a great extent in the coming few years; for instance, material handling in narrow paths to different heights etc. A two-wheeled system is essentially unstable but can be stabilized independently in the same manner as a human. Therefore, a two-wheel system has mobility that is more natural which allows it to interact with humans.

The mechanical structure of a robot with only two driving wheels is similar to an inverted pendulum (D'Andrea and Earl 2005). An inverted pendulum system is an under-actuated mechanical system and inherently open loop unstable with highly non-linear dynamics. It is thus a perfect test-bed for the design of a wide range of classical and contemporary control techniques. It has wide ranging applications from robotics to space rocket guidance systems. The concept of balancing a robot is based on the inverted pendulum model. This model has been widely used by researches worldwide in the design and control of wheeled, legged robots, etc. (Formal and Martynenko, 2005). The inverted pendulum problem is common in the field of control engineering. The uniqueness and wide application of technology derived from this unstable system has drawn interest of many researches and robotics enthusiasts around the world. In recent years, researchers have applied the idea of a mobile inverted pendulum model to various problems, such as designing walking gaits for humanoid robots, robotic wheelchairs and personal transport systems (Kim, et al., 2005). The type of intelligent robot proposed in this work is a mobile robot with a two wheeled inverted pendulum. This design is chosen because its mechanism has an inherently clumsy motion for stabilizing the robot's body posture. The robot has a body with two wheels for moving in a plane and a head similar to a human head for controlling the motion. Two independent driving wheels are used for position control and for fast motion in a plane without casters (Kim *et al*, 2005).

Several kinds of wheels can be attached to the wheeled mobile robot, but they fall into one of two categories: driving wheels and auxiliary wheels. The driving wheels are rotated to allow the robot to move when torque is applied to the axles. The auxiliary wheels merely ease the movement of the robot and enable its body to be suspended when no torque is applied to the axles. By removing the auxiliary wheels, the number of wheels attached on the robot would be reduced. Moreover, the mechanical characteristics of the robot would be completely altered because no elements could suspend and balance the robot's body except for the driving wheels. That is, the robot would have to move and balance its body with only two driving wheels (Kim, *et al*., 2005).

Literature review recorded that most of the work carried out concentrated on developing the control algorithms required to keep the two-wheeled inverted pendulum robot balanced. Other contributions discussed the dynamics of the system. All these works considered a fixed position for the load, which is mainly the global mass of the robot or the rod attached to the axle of the driving wheels. However there is no evidence of any work considering the idea of dealing with an IB attached payload undergoing a linear motion along the IB. This paper presents a two wheeled robot with an additional degree of freedom along its intermediate body to provide a payload with the ability to reach different heights. The payload is considered to simulate the weight of a person on a wheelchair or an object to be handled to different heights if a manual handling task is given to the robot. The work presented in this paper considers challenging control solutions for balancing a two-wheeled robotic machine with changing position and size of the load.

II. SYSTEM DESCRIBTION

The two-wheeled robotic vehicle considered in this work comprises a rod on an axle incorporating two wheels as described in Figures (1) and (2). The robot is powered by two DC motors driving the vehicle wheels and a linear actuator connecting the two parts of the IB which allows the attached payload to move up and down according to a pre described motion scenario. A reference Cartesian coordinate frame designated as $OXYZ$ attached to the axle connecting the wheels with its origin located at the vehicle centre point O as shown in Figures 1 and 2 is used for the angular and translational motion of the vehicle. The $Z-$ axis points vertically upward, the Y axis is parallel and coincides with the wheels axle, and the axis is determined according to the right-hand rule in the rectangular coordinate system. The IB is considered balanced if it coincides with the positive $Z-$ axis. Partial angular deviation from the Z axis causes an imbalance in the vehicle with a tilt angle θ_P around the X axis.

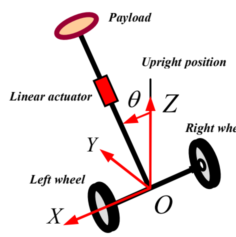

Fig. 1 Two-wheeled vehicle with an extended rod

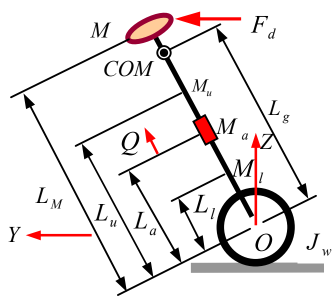

Fig. 2 Positions of vehicle main parts and *Com*

A. Vehicle Lagrangian dynamics

Using Lagrange formulation, the following dynamic equation can be expressed for the system:

$$\frac{dq}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \tag{1}$$

where $L = T_v - V_v$, is the Lagrangian function. The generalized coordinates of the system are chosen as $q_i = [Y \quad \theta \quad Q]^T$ and the generalized force is expressed as $Q_i = [F_c \quad F_d \quad F_a]^T$, Y is the linear displacement of the vehicle, θ is the angular displacement of the IB, Q is the linear displacement of the payload along the IB, F_c is the driving force of the wheels, F_d an external disturbance force and F_a is the linear actuator pushing force. Where the system kinetic and potential energies can be expressed as follows:

$$\begin{aligned} T_v &= \frac{1}{2}\left(M_c + M_{eq}\right)\dot{Y}^2 + \frac{1}{2}\left(M_u + M\right)\dot{Q}^2 \\ &+ \left[C + \left(M_u + M\right)Q\right]\dot{Y}\dot{\theta}\cos\theta \\ &+ \frac{1}{2}\left(C_{22} + C_{20}Q + C_{21}Q^2\right)\dot{\theta}^2 \end{aligned} \tag{2}$$

$$V_v = \left[C + \left(M_u + M\right)Q\right] g \cos\theta \tag{3}$$

B. Vehicle dynamic equations

Manipulating the above expressions yields the following three non-linear differential equations describing the vehicle dynamics alongside the driving forces and the external applied force as:

$$\begin{aligned} &\left(M_c + M_{eq}\right)\ddot{Y} - \left[\left(C + C_9\right)\cos\theta + C_9 Q\right]\ddot{\theta} - \left(C + C_9\right)\dot{\theta}^2\sin\theta \\ &+ C_9\left(\dot{Q}\dot{\theta} + \dot{Q}\cos\theta - Q\dot{\theta}\sin\theta\right) = F_c \end{aligned} \tag{4}$$

$$\begin{aligned} &\left(C_{22} + C_{20}Q + C_{21}Q^2\right)\ddot{\theta} + \left(C_{20} + 2C_{21}Q\right)\dot{Q}\dot{\theta} \\ &+ \left(C + C_9\left(Q + \cos\theta\right)\right)\ddot{Y} + C_9\left(Q - 1\right)\dot{Y}\dot{\theta}\sin\theta \\ &+ C_9\left(\dot{Q}\dot{Y} + \dot{Q}\cos\theta - Q\dot{\theta}\sin\theta\right) - \left(C + C_9 Q\right)g\sin\theta = F_d \end{aligned} \tag{5}$$

$$\left(M_u + M\right)\ddot{Q} - \frac{1}{2}\left(C_{20} + 2C_{21}Q\right)\dot{\theta}^2 - \left(M_u + M\right)\left(\dot{\theta}\dot{Y} - g\right)\cos\theta = F_a \tag{6}$$

III. CONTROL STRATEGY

For the stabilization process of the vehicle, three controllers are developed, as shown in Figure (3). Two of them are used for balancing the vehicle; the first to control the tilt position of the IB and the other for linear position of the vehicle. The third controller is developed to control the linear displacement of the payload along the IB. The control algorithm is implemented using a conventional PID controller in order to test the model. The system inputs are the driving force F_c, the linear actuator force F_a and a disturbance force F_d.

A. Genetic Algorithm Formulation

Genetic Algorithms (GA's) are stochastic global search methods that mimic the process of natural evolution. GA, as introduced by Holland (1975), is an optimization method founded on the principles of natural selection and population genetics. GA has been used in various applications such as signal processing, robotics, active noise cancellation, system identification and modeling, adaptive control, engineering designs, planning and scheduling, and pattern recognition (Chipperfield et al., 2002). Genetic algorithm is an iterative optimization procedure. In the absence of any knowledge of the problem domain, GA begins its search from a random population of solutions. Then a set of biologically inspired operators namely, reproduction, crossover and mutation are applied to update the solutions and this process repeats to a predefined number of times or until a particular target is achieved. The working principle of a GA is shown in the flowchart in Figure (4). The convergence criterion of a GA is a user-specified condition e.g. the maximum number of generations or when the string fitness value exceeds a certain threshold.

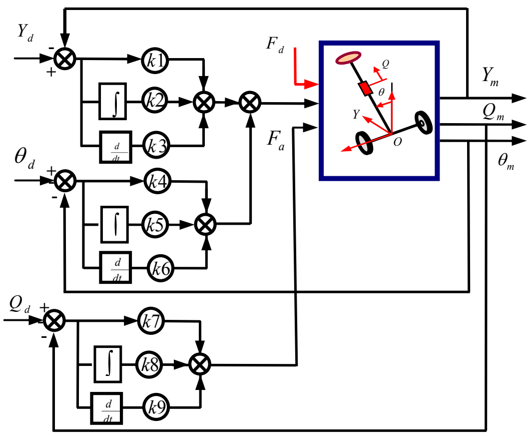

Fig. 3 Schematic description of the control scheme

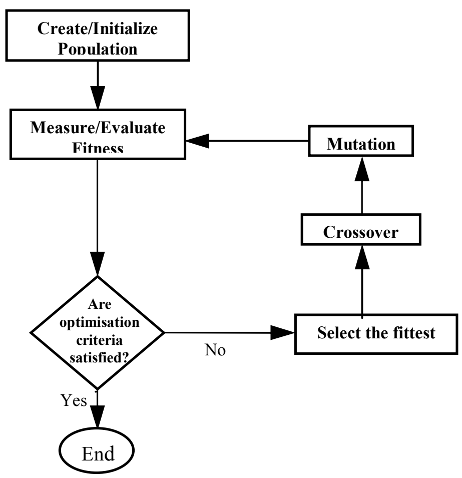

Fig. 4 Graphical Illustration of the Genetic Algorithm process

To optimize the performance of the PID controlled system, the PID gains of the system are adjusted to minimize a certain performance index which is calculated over the entire simulation time. The performance index used in this work is the mean square error (MSE). The purpose of GA optimization process is to optimise the controller parameters based on minimizing the mean squared error for the following objective functions:

$$Obj_1 = \min\left\{\frac{1}{N}\sum_{i=1}^{N}(\theta_d - \theta_m)^2\right\} \qquad (7)$$

$$Obj_2 = \min\left\{\frac{1}{N}\sum_{i=1}^{N}(Y_d - Y_m)^2\right\} \qquad (8)$$

$$Obj_3 = \min\left\{\frac{1}{N}\sum_{i=1}^{N}(Q_d - Q_m)^2\right\} \qquad (9)$$

where θ_d is the desired vehicle energy, θ_m is the measured energy, Y_d is the desired vehicle position, Y_m is the measured vehicle position, Q_d is the desired payload position, Q_m is the actual payload position and N is the number of sample values over the entire simulation period. The parameters of the three PID controllers k_1, k_2, k_3, k_4, k_5, k_6 k_7 k_8 and k_9 are tuned based on minimising the above objective functions.

TABLE 1 Minimum and maximum PID gains

	Minimum	***Maximum***
k_1	**5000**	**7000**
k_2	**70**	**120**
k_3	**1200**	**1500**
k_4	**0**	**0.5**
k_5	**0**	**1**
k_6	**1100**	**1500**
k_7	**15000**	**25000**
k_8	**30000**	**40000**
k_9	**3000**	**7000**

TABLE 2 Optimised PID gains for different objective functions

	Obj_1	Obj_2		Obj_3
	$N_{var}=50$	$N_{var}=20$	$N_{var}=60$	$N_{var}=40$
	$Gen=100$	$Gen=200$	$Gen=100$	$Gen=100$
	***MSE*=0.15581**	***MSE*=0.0051313**	***MSE*=0.00498**	***MSE*=0.00043**
k_1	**5002.3**	**6972.7**	**6975.8**	**5851.9**
k_2	**100.83**	**71.66**	**115.55**	**72.517**
k_3	**1222.8**	**1202.4**	**1203.5**	**1433.4**
k_4	**0.3551**	**0.25146**	**0.14183**	**0.46738**
k_5	**0.05737**	**0.16911**	**0.10888**	**0.49907**
k_6	**1168.1**	**1102.4**	**1103.9**	**1104.2**
k_7	**24619**	**17654**	**15195**	**24956**
k_8	**38678**	**36487**	**39116**	**39917**
k_9	**6922.1**	**3354.6**	**3177.7**	**6999.6**

III. ANALYSIS AND SIMULTION RESUILTS

A. Vehicle and payload prescribed motion

The vehicle is considered to undergo the following scenario during its motion, as shown in Figure (5): starting from rest to a distance of 0.4 m with a constant speed of 2cm/s, keep stationary for 10 seconds, moving further 10cm with a speed of 1 cm/s until fixed again for 20 seconds. Finally, it is allowed to repeat the schedule in a reverse direction until it comes back to rest again.

While the vehicle undergoes the previous mentioned motion, the linear actuator, with a stroke limited to 50cm, is allowed to activate the attached payload. This corresponds to a scenario of the behavior of a disabled person sitting on a wheelchair. The prescribed payload motion, as shown in Figure (6), should comprise periods of motion for performing certain tasks or handle items, stationary periods to get a decision and a region of coming down to the rest position. The speed of the payload is limited to 2cm/s and 6cm/s during the moving up periods while it is going back to rest with 3.3 cm/s.

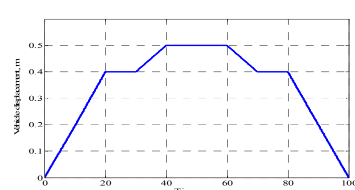

Fig. 5 Prescribed vehicle linear displacement

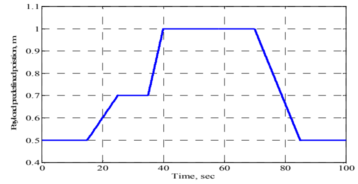

Fig. 6 Prescribed motion of the payload

The tuning of the PID gains is based on minimizing three objective functions consecutively. The simulation is carried out with different number of individuals and for different number of generations. For all the objective functions used, the GA process was initialized with randomly generated number of individuals. Each individual had 9 strings which represent the gains of the three PID controllers; k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8 and k_9. The ranges for those gains are shown in Table 1. The min and max range for each gain has been identified based on knowledge about the system performance gained during the simulation using manual tuning.

Optimization process of the controller gains is carried out using each objective function at a time. The idea is to investigate which way will lead to a better performance of the entire vehicle by minimizing the performance index for the three system outputs. The process has been carried out for one run for objective functions 1 and 3. However it was repeated for objective function 2 with different number of individuals and for different number of generations. The optimized PID gains are illustrated in Table 2.

System performance thus achieved is shown Figures 7, 8, 11 and 12. Minimizing objective function 1 tends to decrease the overshoot in the IB angular position by around 30% less than the case using objective 2 and 3 and decrease the vibration of the IB, see Figures 7(b), 8(b), 11(a) and 12(a). . However, despite the improvement in the IB percent overshoot there was a large overshoot in the vehicle displacement, Figure 7(c), compared to the case when using objective functions 2 and 3, Figures 8(c), 11(b) and 12 (b).

The amount of energy consumed by the vehicle for a complete one cycle is illustrated in Figures 9, 10, 13 and 14. The vehicle will still keep a level of energy even for periods of rest or when achieving balance at the upright position. This level of energy is mainly due to the motion of the payload as a sort of kinetic energy or even due to the instantaneous position of the payload caused by the linear actuator as a form of potential energy.

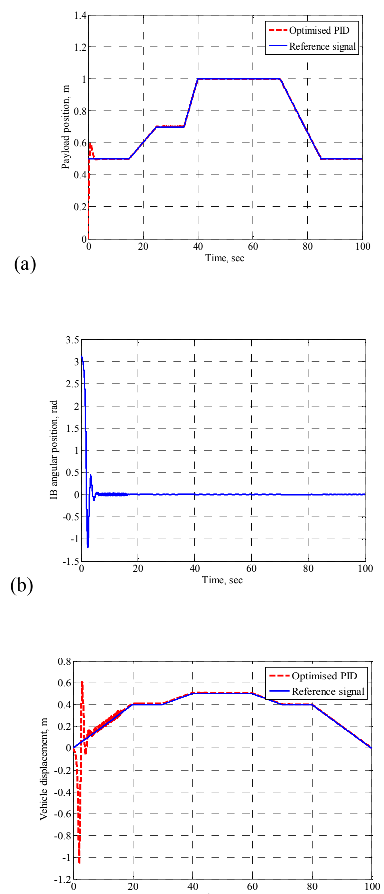

Fig. 7 System performance, objective function **1** (50 individuals, 100 generations)

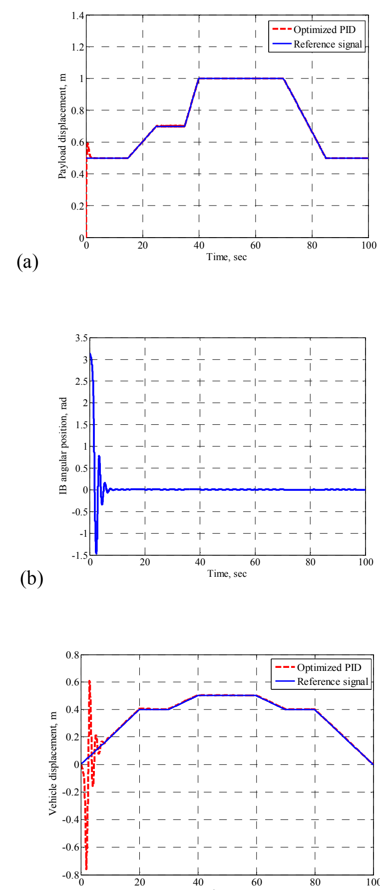

Fig. 8 System performance, objective function **2** (20 individuals, 200 generations)

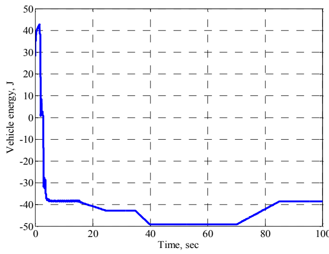

Fig. 9 System energy, objective function **2** (60 individuals, 100 generations)

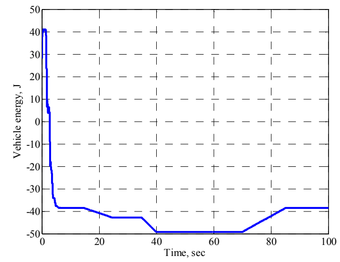

Fig. 10 System energy, objective function **2** (60 individuals, 100 generations)

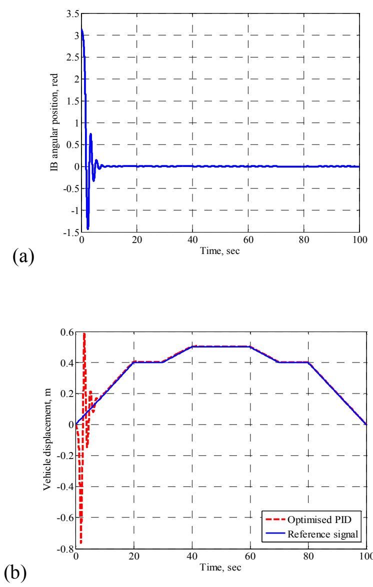

Fig. 11 System performance, objective function **2** (60 individuals, 100 generations)

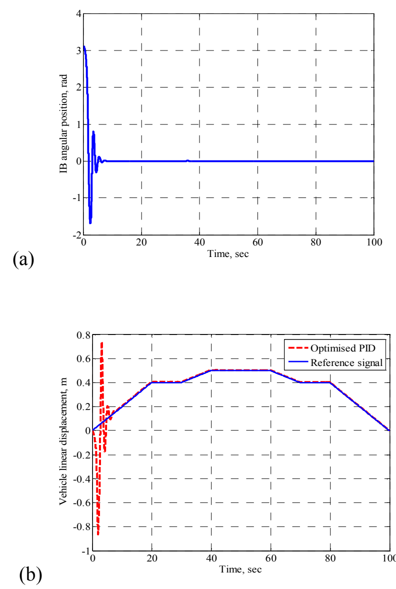

Fig. 12 System performance, objective function **1 (**40 individuals, 100 generations)

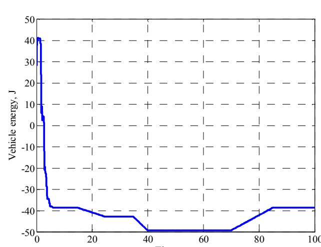

Figure 13 System energy, objective function 2 (60 individuals, 100 generations)

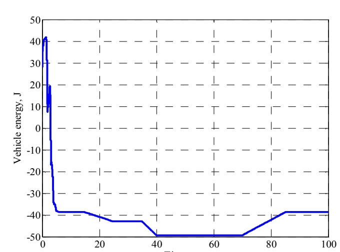

Fig. 14 System energy, objective function **3** (40 individuals, 100 generations)

CONCLUSIONS

The design and implementation of a PID control algorithm utilizing genetic algorithm as an optimization technique for a novel design of a two-wheeled vehicle with an extended rod has been presented. This novel design offers the system an additional degree of freedom in a vertical direction which allows the vehicle to be utilized for applications like handling objects to different heights in industrial applications. The idea of a vehicle with such additional motion is considered to be a test base for application of wheelchairs on two wheels. Energy analysis and feedback control schemes have been developed to balance the vehicle. Simulation results have been presented to address the effect of applying a disturbance force on the system with an activated payload along the IB. The dynamic behavior of the system has been presented alongside the level of energy consumed for achieving the required performance.

Appendix

$C_1 = M_l L_l^2 + M_a L_a^2$, $C_2 = L_l$, $C_3 = L_a = 2L_l$, $C_4 = 2L_l + L_u$,

$C_5 = 2L_l + 2L_u$, $C_6 = M_l L_l + M_a L_a$, $C = C_6 + M_u C_4 + MC_5$,

$C_7 = M_l / M_{eq}^2$, $C_8 = C_6 + M_u C_4 + MC_5 - M_{eq} L_l$, $C_9 = M_u + M$,

$C_{10} = M_a / M_{eq}^2$, $C_{11} = C_6 + M_u C_4 + MC_5 - M_{eq} L_a$,

$C_{12} = M_u / M_{eq}^2$, $C_{13} = C_6 + M_u C_4 + MC_5 - M_{eq} C_4$, $C_{14} = M / M_{eq}^2$,

$C_{15} = -C_6 - M_u C_4 - MC_5 + M_{eq} C_5$,

$C_{16} = C_7 C_8^2 + C_{10} C_{11}^2 + C_{12} C_{13}^2 + C_{14} C_{15}^2$,

$C_{17} = 2C_7 C_8 C_9 + 2C_9 C_{10} C_{11} + 2C_{12} C_{13} \left(M_u + M - M_{eq}\right)$,
$+ 2C_{14} C_{15} \left(M_{eq} - M_u - M\right)$

$C_{18} = C_7 C_9^2 + 2C_9^2 C_{10} + C_{12} \left(M_u + M - M_{eq}\right)^2 + C_{14} \left(M_{eq} - M_u - M\right)^2$

$C_{19} = C_{16} + M_u C_4^2 + MC_5^2$, $C_{20} = C_{17} + 2M_u C_4 + 2MC_5$

$C_{21} = C_{18} + C_9$, and $C_{22} = C_1 + C_{19}$.

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