The Unused Capacity Minimization in WDM Mesh Networks with Sharing Optical Path Protection by Finite Differences

St. Mylonakis, IEEE

***Abstract*- In this paper, the shared protection method will be examined with the linear function method and the accurate polynomial function method of the finite differences. The merit of these methods is the successful solving of the problems of protected networks and they can used to verify each other. Then the problem of the unused available network capacity minimization is approached by four method combination (both finite differences methods are used in three of four methods). First and second are ILP methods, third is statistic one and forth is a simplex algorithm one.**

Key words: finite differences, linear, polynomial, shared protection, unused network capacity minimization

I. INTRODUCTION

The WDM optical mesh networks are high capacity telecommunication networks based on optical technologies. Research has been done [1]-[18] in relation to the methods and the problems associated with planning, protection and restoration of optical networks as well as the resource optimization using several mathematics methods. The most used methods are special approaches based on ILP .In [1], there are finite difference issues . In [2], the book is an attempt to make the modeling and analysis of system performance more methodical and more realistic. In [3], there are optimization issues. The network survivability has been extensively studied. There are several approaches to ensure fiber network survivability [4] and [5]. In [6] the authors write about the evolution of the OTN from operators view. In [7] a practical approach to operating survivable WDM networks is provided when the network operation is under dynamic traffic. In the [8] and [9] address issues in designing a survivable optical layer. In [10] a mesh based hybrid OMS / OCh protection /restoration scheme is suggested. In [11], the authors suggest a dynamical bandwidth distribution for protection in IP over WDM networks. In [12], a preplanned local repair restoration for Optical Transport Network is suggested. In [13] the authors propose a strategy for Protection and Restoration of Optical Paths in WDM Backbone Networks for Next Generation Internet Infrastructures. The shared path protection problem is examined in the papers [14], [15] and [16]. The common characteristic of these papers is to use the shared protection path to improve the resource efficiency resulting from backup sharing. In [14], a new shared path protection is suggested supported by dynamic provisioning of restorable bandwidth. In [15], two link disjoint paths, a dedicated working path and a shared protection path are computed , for an incoming light path request based on the current network state but the protection approaches to optimize the resource utilization for a given traffic matrix, do not apply because lightpath requests come and go dynamically. In [16], the approach of path protection is examined, its wavelength capacity requirements, the routing and wavelength assignment of the primary and backup paths, as well as the protection switching time and the susceptibility of these schemes to failures. In [17], the approach of path protection is examined using shared spare lightpaths. In [18] deals with the modeling and simulation and gives practical advices for network designers and developers.

Manuscript received June 28, 2011.

St.T. Mylonakis is with the National University of Athens, Athens, Attica, GREECE (corresponding author to provide phone: 00302108814002; fax: 00302108233405; e-mail: smylo@ otenet.gr).

The network topology and other parameters are known as WDM and optical fiber capacity, one optical fiber per link with an extension to a 1+1 fiber protection system. So this network is characterized by one working fiber per link, edges of two links, links of two optical fibers, one for working and one for protection. The connections are lightpaths originating in the source nodes and terminating at the destination nodes proceeding from preplanned optical working paths. Additionally, the same numbers of optical paths are preselected for the preplanned fully disjoint backup paths, (1:1 sharing protection connection). Thus the connections that have been set up are protected. The connections of the same node pair by same preplanned optical paths form a connection group along the network. The sharing protection of the connection groups is done by preplanned optical protection paths with the reservation of a suitable number of wavelengths per link along the network. The problem solution is to calculate and minimize the final unused available capacity of the network that depends of the sizes of the connection groups (it is assumed that each connection group size is one at least). A table is used and contains the number of the node pairs, the node pairs with their preplanned working and protection paths .The role of the Difference Calculus is in the study of the Numerical Methods. These Numerical Methods are solved by computer. The subject of the Difference Equations is in the treatment of discontinuous processes. The network final unused available capacity is expressed as a difference equation because the final unused available capacity of the individual working optical fibers is also a difference equation. The reduction of the available capacity of each working optical fiber is a discontinuous process when connection groups of several sizes pass through it. Each difference equation is revealed by two methods, the linear function method and the polynomial function method. These methods include accurate and arithmetical methods to study the optical networks and their problems, solving the same problem so they can used to confirm each other.

This paper is broken down in the following sections: Section II shows how the finite differences are used for each optical fiber and illustrates the optical fiber residual available capacity; Section III describes the problem and provides a solution, the algorithm detailed description, an example, the execution time, the proposals and a discussion; Section IV draws conclusions and finally ends with the references.

II. THE OPTICAL FIBER AND THE FINITE DIFFERENCES

Before studying finite differences and their use in optical WDM mesh networks survivability, it is necessary to provide a short presentation of the finite differences computation. Let's assume that y_1, y_2, …, y_n is a sequence of numbers in which the order is determined by the index *n*. The number *n* is an integer and the y_n can be regarded as a function of *n*, an independent variable with function domain the natural numbers and it is discontinuous. Such a sequence shows the available capacity reduction of a telecommunication fibre network link between two nodes when the telecommunication traffic of 1, 2,…, *n* source destination node pairs pass through. It is assumed that the telecommunication traffic unit is the optical channel that is one wavelength (1λ). The telecommunications traffic includes optical connections with their protections. The total connections of a node pair form its connection group. The first order finite differences represent symbolically the connection group of each node pair that passes through a fiber. This connection group occupies the corresponding number of optical channels and it is the bandwidth that is consumed by connections of a node pair through this fiber. The first order finite differences are used to represent the connection groups in optical channels of the node pairs that pass through an optical fiber .An equation of the first order finite differences gives the available capacity of an optical fiber network link when a connection group passes through it. This available capacity is provided for the connection groups of the other node pairs that their connections will pass through this optical fiber. Thus the total unused available capacity of each network optical fiber is calculated after *n* connections groups pass through it. This could be written with two methods, the linear function method and the polynomial function method. So these methods could be used to check each other. Table 1 gives a short presentation of the computation of the finite differences for a given link that can be arranged quite simply. At the first column the node pairs are presented. At the second column the indexing or the numbering of the node pairs is presented. At the third column, the available capacity that offered to connection groups is presented. At the forth column, the number of the connection groups that pass through. At the fifth column, the differences between successive connection groups are represented. At the sixth column, the differences between the successive elements of the column five and called high order differences are presented.

TABLE 1
A SHORT PRESENTATION OF THE FINITE DIFFERENCE COMPUTATION

Node Pair	Node pair index	Available capacity connection group	Number of the optical connection group	Variation of the optical differences number	Higher order differences number
(S_1,D_1)	1	y_1			
			Δy_1		
(S_2,D_2)	2	y_2		$\Delta^2 y_1$	
			Δy_2		$\Delta^3 y_1$
(S_3,D_3)	3	y_3		$\Delta^2 y_2$	
			Δy_3		
(S_4,D_4)	4	y_4			
.....					
(S_{n-1},D_{n-1})	n-1	y_{n-1}			
			Δy_{n-1}		
(S_n,D_n)	n	y_n		$\Delta^2 y_{n-1}$	
			Δy_n		
(S_{n+1},D_{n+1})	n+1	y_{n+1}			

The unused available capacity in the linear function method of an optical fiber link after *n* connection groups have passed through it corresponding to the communication between *n* source destination node pairs is the following.

$$y_{n+1}= y_1 - \sum_{j=1}^{n} \Delta y_j - u_{max} \quad (1)$$

y_{n+1} is the unused available capacity in optical channels (wavelengths) of the optical fiber network link for the n+1 node pairs.

y_1 is the available working capacity in optical channels (wavelengths) of the optical fiber for the first node pair. It gives the installed capacity and it is a boundary condition.

The second term of the right part is the sum of *n* first order finite differences and it is the sum of all optical channels of *n* nodes pairs that pass through this optical fiber.

u_{max} is the reserved spare sharing protection capacity in optical channels (wavelengths) of the optical fiber network link.

Second and higher order finite differences are used to represent other variations.

It is also valid

$$\sum_{j=1}^{n} \Delta y_j = \sum_{l=1}^{n} a_l * x_l \quad (2)$$

Then the equation of the available capacity and is also written

$$y_{n+1}= y_1 - \sum_{l=1}^{n} a_l * x_l - u_{max} \quad (3)$$

with a_l is a coefficient that takes the value one if the node pair (l) passes all its connections from this fiber and zero (0) if no passes.

x_l is the total of the connections of each node pair (l). This is the connection group and it is called connections group size.

n the total number of the node pairs.

The general form of a polynomial function that gives the unused available capacity of the optical fiber network link after the serving n connection groups, is as follows

$$y_{n+1}= \sum_{r=0}^{n} \alpha_r *(n +1)^{r} \quad (4)$$

The assessment of the polynomial function coefficients (α_r) done with the values that the polynomial function represents for 1,2,…, n, n +1 .The values of the function y_{n+1} for each n must be integral because each value represents optical channels. The spare sharing capacity impacts on the available capacities.

Notes on the polynomial function.

-When no connection group passes through a fiber, the polynomial function is constant.

-When only one connection group passes through an optical fiber the polynomial function has the first degree.

-When only a group of two connections pass through an optical fiber the polynomial function has the second degree, etc.

-The degree of the polynomial function of an optical fiber depends of the number of the connection groups that pass through it.

-Two polynomial functions with the same available capacity have different coefficients when the order and the size of the same number connection groups are different.

-The polynomial function of an optical fiber is different when there is full or partial servicing of the connection groups that pass through it.

In the table 2, the analytical form of the equation 4 is presented. The value of the function has high accuracy of 15 decimal digits. This method is an *accurate* one.

TABLE 2
ANALYTICAL PRESENTATION OF EQUATION 4

n	y_{n+1}	r	α_r	Polynomial form, $\Sigma\ \alpha_r*(n+1)^r$
0	y_{0+1}	0	α_0	$\alpha_0*(0+1)^0$
1	y_{1+1}	0, 1	α_0, α_1	$\alpha_0*(1+1)^0+\alpha_1*(1+1)^1$
2	y_{2+1}	0, 1, 2	α_0, α_1, α_2	$\alpha_0*(2+1)^0+\alpha_1*(2+1)^1+\alpha_2*(2+1)^2$
3	y_{3+1}	0, 1, 2, 3	α_0, α_1, α_2, α_3	$\alpha_0*(3+1)^0+\alpha_1*(3+1)^1+\alpha_2*(3+1)^2+\alpha_3*(3+1)^3$
.	....	..		....
n	Y_{n+1}	0, 1, 2, 3, ..., n	α_0, α_1, α_2, α_3,.., α_n	$\alpha_0*(n+1)^0+\alpha_1*(n+1)^1+\alpha_2*(n+1)^2+\ldots+\alpha_n*(n+1)^n$

There are systems of n+1equations with n+1unknown coefficients. The values of the coefficients depend of the number of the connection groups and the connections of each connection group. It is ought to the Table 1 column 4. This method is accurate because only one factor is added to new equation when the polynomial degree increases. The equations, (1) and (4) must be greater or equal to zero, for full servicing all connection groups that pass through an optical fiber. So the number of connections on each link is bounded.

The polynomials that calculate the available capacity of each optical link for the accurate method for all possible values up to four and for the following cases are represented. The number, the order and the size of $\underline{\Delta y_{i,j}}$ are critical. The WDM system capacity is 30 λ. So for the accurate method it is written.

-If no one-connection group passes through an optical link the polynomial function is constant, etc.

$\underline{y_{j,0+1}=\alpha_0*(0+1)^0}$

$y_{j,0+1}=30.$

-If only one-connection group passes through an optical link the polynomial function is of the first degree.

$\underline{\Delta y_{i,1}, \quad y_{i,1+1}=\alpha_0*(1+1)^0+\alpha_1*(1+1)^1}$

1 $\quad y_{i,1+1}=30*(1+1)^0-(1/2)*(1+1)^1=29$

2 $\quad y_{i,1+1}=30*(1+1)^0-(2/2)*(1+1)^1=28$

3 $\quad y_{i,1+1}=30*(1+1)^0-(3/2)*(1+1)^1=27$

4 $\quad y_{i,1+1}=30*(1+1)^0-(4/2)*(1+1)^1=26$

-If only two-connection groups pass through an optical link the polynomial function is of the second degree.

$\underline{\Delta y_{i,1}, \Delta y_{i,2}, y_{i,1+1}=\alpha_0*(2+1)^0+\alpha_1*(2+1)^1+\alpha_2*(2+1)^2}$

1 , 1, $y_{i,2+1}=30*(2+1)^0-0.5*(2+1)^1-5.55555555555429E\text{-}2*(2+1)^2=28$

2 , 1, $y_{i,2+1}=30*(2+1)^0-1.0*(2+1)^1-0.00000000000000E\text{+}0*(2+1)^2=27$

1 , 2, $y_{i,2+1}=30*(2+1)^0-0.5*(2+1)^1-1.66666666667420E\text{-}1*(2+1)^2=27$

3 , 1, $y_{i,2+1}=30*(2+1)^0-1.5*(2+1)^1+5.55555555555429E\text{-}2*(2+1)^2=26$

2 , 2, $y_{i,2+1}=30*(2+1)^0-1.0*(2+1)^1-1.11111111111086E\text{-}1*(2+1)^2=26$

1 , 3, $y_{i,2+1}=30*(2+1)^0-0.5*(2+1)^1-2.77777777777828E\text{-}1*(2+1)^2=26$

4 , 1, $y_{i,2+1}=30*(2+1)^0-2.0*(2+1)^1+1.11111111111086E\text{-}1*(2+1)^2=25$

3 , 2, $y_{i,2+1}=30*(2+1)^0-1.5*(2+1)^1-5.55555555555429E\text{-}2*(2+1)^2=25$

2 , 3, $y_{i,2+1}=30*(2+1)^0-1.0*(2+1)^1-2.22222222221720E\text{-}1*(2+1)^2=25$

1 , 4, $y_{i,2+1}=30*(2+1)^0-0.5*(2+1)^1-3.88888888886870E\text{-}1*(2+1)^2=25$

4 , 2, $y_{i,2+1}=30*(2+1)^0-2.0*(2+1)^1+0.00000000000000E\text{+}0*(2+1)^2=24$

3 , 3, $y_{i,2+1}=30*(2+1)^0-1.5*(2+1)^1-1.66666666667420E\text{-}1*(2+1)^2=24$

2 , 4, $y_{i,2+1}=30*(2+1)^0-1.0*(2+1)^1-3.33333333333485E\text{-}1*(2+1)^2=24$

e.t.c

The fiber spare capacity (u_{max}) is presented below. A protection sub-network protects each optical fiber. A number of working connection groups pass through an optical fiber as well as a number of wavelengths are reserved for the protection of another number of working connection groups. But these working connection groups have protection lightpaths pass through other optical fibers. So the protection subnetwork of an optical fiber is the network that is formed by the reserved wavelengths of the protection lightpaths of these connection groups. The total number of shared protection wavelengths is calculated assuming the wavelengths sharing. If any optical fiber is not used for protection subnetwork, this optical fiber does not need any spare protection capacity.

III.THE PROBLEM AND ITS SOLUTION

A. The problem

The network topology and other parameters are known as WDM and optical fiber capacity, one optical fiber per link with an extension to a 1+1 fiber protection system. So this network is characterized by one working fiber per link, edges of two links, links of two optical fibers, one for working and one for protection. The connections are lightpaths originating in the source nodes and terminating at the destination nodes proceeding from preplanned optical working paths. Additionally, the same number of optical paths is preselected for the preplanned fully disjoint backup paths, (1:1 sharing protection connection). Thus the connections that have been set up are protected. The connections of the same node pair by same preplanned optical paths form a connection group along the network. The sharing protection of the connection groups is done by preplanned optical protection paths with the reservation of a suitable number of wavelengths per link along the network. The problem solution is to calculate and minimize the final unused available capacity of the network that depends of the sizes of the connection groups (it is assumed that each connection group size is one at least). A table is used and contains the number of the node pairs, the node pairs, the node pairs sizes with their preplanned working and protection paths. The table 3 shows the symbols that used in this paper.

B. The formulation

A difference table (table 1) is calculated for each optical fiber and the problem is solved using two methods, the first is the linear function one and the second is the accurate polynomial function one.

The formulation of the *linear function* method is presented below. The unused available capacity in optical channels for each optical fiber when n(i) groups of working optical connections pass through it, is equal to the unused available capacity in optical channels that is offered for the first group of working optical connections minus the total number of n(i) groups of working optical connections that pass through it minus the reserved sharing protection capacity. The residual unused available capacity of all optical fibers is written as a column matrix. The equation of the linear function method is written as

$$Y_2=Y_1-A*Xn-Umax \qquad (5)$$

A is a matrix that shows the active optical fiber network links (*2p*) from which the (*n)* working connection groups pass so its dimension is (2p x n) and its element are the coefficients $a_{i,j}$, *Umax* is a column matrix consisting of elements related to the number of the reserved wavelengths for sharing protection of each link, Y1 is the matrix with the installed capacity of each optical fiber network link and Y_2 the matrix which has elements the unused available capacity of each optical fiber network link. When all connections have been set up then each element of Y_2 must be greater than or equal to zero. In other cases some connections are not set up.

The equation of the polynomial function method is similar to that of the equation (4) but for all network fibers there are two column matrices, the left one that equals with the right one. When all connections have been set up, each element of the left column matrix must be greater or equal to zero. In other cases some connections are not possible.

The reserved wavelengths for *sharing protection* are common for both function methods (linear and polynomial) and therefore the total *protection capacity* C_{pr} of the sharing protection for all fibers is the following

$$C_{pr} = \sum_{i=1}^{2p} u_{i,max} \qquad (6)$$

The formulation of the linear function method for the total unused available capacity of the network is the following

$$\sum_{i=1}^{2p} y_{i,n(i)+1} = \sum_{i=1}^{2p} y_{i,1} - \sum_{i=1}^{2p} \sum_{j=1}^{n(i)} a_{i,j} * x_j - \sum_{i=1}^{2p} u_{i,max} \qquad (7)$$

$y_{i,n(i)+1}>=0$ and $y_{i,n(i)+1}<= y_{i,1}$, $y_{i,1}>=0$, $a_{i,j}>=0$, $u_{i,max}>=0$ and $u_{i,max}<= y_{i,1}$, $x_j>0$ and , $x_j<= y_{i,1}$

The formulation of the *polynomial function* method for the total unused available capacity of the network is the following

$$\sum_{i=1}^{2p} y_{i,n(i)+1} = \sum_{i=1}^{2p} \sum_{r=0}^{n(i)} \alpha_{i,r} * (n(i)+1)^r \qquad (8)$$

$y_{i,n(i)+1}>=0$ and $y_{i,n(i)+11}<= y_{i,1}$,n(i)>0

The optimization is realized for both methods so that to minimize the total network unused available capacity

$$\text{Min}\left(\sum_{i=1}^{2p} y_{i,n(i)+1}\right) \qquad (9)$$

The number of connections on each link is bounded

$$\sum_{j=1}^{n(i)} a_{i,j} * x_j + u_{i,max} <= y_{i,1} \quad 1<=i<=2p \qquad (10)$$

or

$$\sum_{r=0}^{n(i)} \alpha_{i,r} * (n(i)+1)^r <= y_{i,1} \quad 1<=i<=2p \qquad (11)$$

The demand for the primary connections between each node pair is satisfied as

$$d_j = \sum_{i=1}^{2p} a_{i,j} * x_j \qquad (12)$$

The other boundary conditions are the following:

$y_{i,n(i)+1}>=0$ and $y_{i,n(i)+1}<= y_{i,1}$, $y_{i,1}>=0$, $a_{i,j}>=0$, $u_{i,max}>=0$ and $u_{i,max}<= y_{i,1}$, $x_j>0$ and , $x_j<= y_{i,1}$

The decision variable is x_j and the optimization of ILP is done after the calculation of the linear and the polynomial methods.

TABLE 2

THE SYMBOLS OF THIS PAPER

SN	Symbol	Commends
1	q	The node number
2	p	The edge number
3	G(V,E)	The network graph
4	V(G)	The network node set
5	E(G)	The network edge set
6	2p	The number of working and backup fiber for 1+1 line protection
7	n	The number of source – destination nodes pairs of the network
8	Xn	Column matrix with dimension (nx1) and elements the connection group sizes of the corresponding source-destination nodes pairs
9	n(i)	The number of the connection groups that passes through the fiber (*i*) and means that each fiber has different number of connection groups pass through it
10	k	The number of the wavelengths channels on each fiber that is the WDM system capacity
11	Y_1	Column matrix (2px1) with elements the installed capacity of fiber network links of the linear function method
12	Y_2	Column matrix (2px1) with elements the unused available capacity of each fiber network link of the linear function method
13	A	Matrix (2p x n) which shows the network active links that corresponding to working fibers
14	$a_{i,j}$	Element of the matrix A and takes the value one if the node pair (j) passes all its primary connections from the fiber (i) and zero (0) if no passes
15	$\Delta y_{i,j}$	First order of finite difference that corresponds to a group of optical connections that pass through the optical fiber ***i*** with serial number ***j*** and valid 1<=i<=2p and 1<=j<=n(i)
16	$y_{i,j}$	Unused available capacity of the optical fiber i that it is offered for optical connections group with serial number j and valid 1<=i<=2p and 1<=j<=n(i)
17	U_{max}	Column matrix (2px1) and elements the spare capacity of each optical fiber that is reserved for sharing protection
18	$u_{i,max}$	Element of the U_{max} column matrix and shows spare capacity of the optical fiber (***i***) that is reserved for sharing protection
19	u_{max}	Spare capacity of one optical fiber that is reserved for sharing protection
20	d_j	Demand of each node pairs
21	$a_{i,r}$	Real number coefficient for the polynomial function method for the link (*i*).
22	Cinst	The total installed capacity
23	Cav	The total available capacity
24	Cw	The total working capacity
25	Cpr	The total protection capacity
26	<=	Means less or equal

C. Detailed description of the algorithms

These algorithms describe the operation of the WDM optical fiber mesh network with 1+1 optical fiber protection, 1:1 optical connection sharing protection by preplanned working and protection paths. TURBO PASCAL is used to program the model. The following algorithms are used in this paper. The algorithm with "Network with failure" is called *simple* algorithm and it is showed at the example case one. The

1st optimization algorithm is the algorithm with the optimization steps. The *2nd optimization* algorithm is the algorithm with the optimization steps but the node pairs are sorted according to the primary path length. The *3rd optimization* algorithm is a statistical method that uses a random number generator at the step 2 and for a larger number of instances. The 4^{th} *optimization* algorithm is a variation of the simplex algorithm. These are showed at the example case two.

First step *Network parameters*

Initially the following data are known, network topology, node number, edge number, link number per edge, working optical fiber number per link, protection optical fiber number per link, wavelength number per optical fiber, optical fiber numbering. This information allows the computer to draw a graph and an OXC is on each vertex of the graph. Each edge corresponds to two links with opposite direction to each other. All fibers have the same wavelength number and all links the same fiber number.

Second step *Connection selections*

In this step, the connection node pairs number, the connection node pair selection for connections and the desired connection group size are done. The preplanned working and the protection optical paths for connections of every node pair are also provided. In this step, the random number generation is activated and gives values to the connection groups of each node pair (*3rd optimization* algorithm). In this step, the 1^{st} and 2^{nd} optimization algorithms set all connection group sizes to 1.

Failure-free Network Phase

Third step *Calculation of reserved wavelengths*

In this step, the protection wavelengths for each optical fiber as well as the total reserved protection wavelengths are calculated because the connection group size, the corresponding backup lightpaths as well as the other network parameters which are needed, are known. The protection wavelengths are calculated for optical connection sharing protection 1:1.

Forth step *Wavelength allocation*

In this step, wavelength allocation is initiated. A working connection starts from the source node and progresses through the network occupying a wavelength on each optical fiber and switch to another fiber on the same or other wavelength by OXC, according to its preplanned optical path up to arrive at the destination node. Simultaneously, for the shared protection method, the protection connection starts from the source node and progresses through the network reserving a wavelength on each optical fiber and switch to another fiber on the same or other wavelength by OXC, according to its preplanned protection optical path up to arrive at the destination node. The number of connections of each node pair is equal to its connection group size. After it, the available capacity is also calculated under both methods of the finite differences. Thus the unused available capacity of the one method is compared to unused available capacity of the other method for one connection.

Fifth step *Presentation of the finite differences*

The total unused available capacity of each optical fiber is calculated and represented under both finite difference methods, i.e. the linear function one and the polynomial function one. Thus the total fiber unused available capacity under the first method is compared to the total fiber capacity available of the other method for all connections.

Sixth step *Results and comparisons thereof*

Having the desired connection group size the total results are computed under each finite difference method that are the total sum of the individual connection group size, the total installed capacity, the total working capacity, the total protection capacity, the total busy capacity and the total unused available capacity. These results of the linear function method are compared to the results of the accurate polynomial function method. The results of the linear method must to be equal to the results of the polynomial method. This network is planned and designed as a strictly nonblocking network, in which it is always possible to connect any node pair, regardless of the state of the network. So all requests for connection are satisfied and form connections. If there is no failure, the algorithm is terminated.

Network with failure Phase

When a failure occurs and a link is cut, the optical fibers of this link are also cut and the optical fiber protection 1+1 and the network topology change. The connection groups that passed through the cut link are also cut and the traffic passes through the preplanned protection paths of other links. The computer is informed of the cut link and modifies suitably the network parameters. The cut optical fiber sets its wavelengths to zero. The connection groups that passing through the cut link set their using wavelengths to zero and through the others to free. The matrix A changes as well as the number of the group size that passing through optical fibers and the algorithm is repeated from the begin calculating new results and after them the algorithm ends.

The optimization problem

The optimization is generated by four methods. In the first method, the optimization (minimization) is obtained modifying the simple algorithm (to subject to the ILP equations) setting all connection group sizes equal to 1 and computes the connection group sizes of the corresponding source-destination nodes pairs that minimize the network unused available capacity (one optimization step). Then these connection group sizes are used to generate the minimized unused available network capacity (second optimization step). This method could generate directly, in one step, the connection group sizes and the minimum unused available capacity so the second step is used for verification. In the second method the node pairs are sorted according the primary path length in hops and the optimization is obtained as the first method. In the third one, the optimization is obtained by statistical method. The unused available network capacity is calculated for a large number of instances. The values of connection group sizes are random with adjusted variable maximum value and a minimum unused available capacity is obtained. In the forth case, the optimization is obtained by a variation of the simplex algorithm for one scenario every time.

D. Example

The network is assumed to be an optical mesh network with the circuit switched or packet switched but packets are gone by preplanned lightpaths as a graph. Each vertex represents the central telecommunications office (CO) with the OXC while each edge represents the two links of opposite direction each other. Each edge link has a couple of optical fibers. All optical fibers have the same capacity as the WDM system. All nodes are identical. The numbers of working and protection connections that pass through each optical fiber are different. The topology of the network is presented by the graph G (V, E). This mesh topology is used because it is a simple, palpable and an analytical example of the finite differences and it is easy to expand to any mesh topology. The vertex set has q=5 elements and the edge set has p=6 elements. Each edge is has two optical links of opposite directions with one working and one protection fiber. Thus there are 2*p=2*6=12 working and 12 protection optical fibers. Connection groups transverse the mesh network. In this example, the linear function method and the accurate polynomial function one are presented. In the *first case*, the problem is solved for *an instance with n=10 of 20 possible* node pairs. These have their order and sizes for each source-destination node pair, their working paths and their protection paths as shown in table 5. In the second case, the optimization is obtained. Figure 1 presents the mesh topology. Table 4 presents the network parameters.

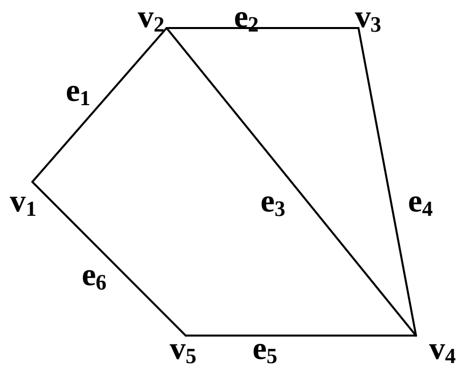

Fig1.The mesh topology of the network

TABLE 4
THE NETWORK PARAMETERS

S/N	Network parameters	Amount
1	**Node number**	**5**
2	**Edge number**	**6**
3	**Working fiber per edge**	**2**
4	**Working fiber per link**	**1**
5	**Network working fiber**	**12**
6	**Protection fiber per edge**	**2**
7	**Protection fiber per link**	**1**
8	**Network protection fiber**	**12**
9	**WDM system capacity**	**30**

It is obvious that the dedicated path protection mechanisms use more than 100% redundant capacity because their lengths are longer than their working paths. The total length of working paths is eighteen, (18) and the total length of protection paths is twenty-four, (24). The algorithm gives that total length of sharing protection paths is nineteen, (19).

In the *first case*, the table 1 (finite difference table) of each fiber is not presented because the number of these tables is six, (6). The higher order finite differences and the number of connection groups that pass through each optical fiber are showed in the following table 6. (Fiber, i) shows the optical fibers. Optical fiber link means the corresponded link of this fiber. The n(i) shows the number of the connection groups that pass through each optical fiber. The $\Delta^{m(i)}y_i$ the order finite differences with m(i)=1, 2, 3, 4, 5 of the fiber i. The intermediate order finite differences are not showed. Fiber 11 has no 3rd order finite difference because its 2nd order finite differences are same. It is obvious that optical fiber nine (9) has the larger difference table.

TABLE 5
THE NODE PAIRS WITH PREPLANNED PATHS AND THE CONNECTION GROUP SIZE

Node pair [Si,Di]	Node pair $[v_i, v_j]$	Working Path	Protection Path	Group size
$[S_1, D_1]$	$[v_1, v_2]$	v_1, v_2	v_1, v_5, v_4, v_2	**1**
$[S_2, D_2]$	$[v_1, v_3]$	v_1, v_2, v_3	v_1, v_5, v_4, v_3	**3**
$[S_3, D_3]$	$[v_1, v_4]$	v_1, v_2, v_4	v_1, v_5, v_4	**2**
$[S_4, D_4]$	$[v_2, v_4]$	v_2, v_4	v_2, v_3, v_4	**4**
$[S_5, D_5]$	$[v_2, v_5]$	v_2, v_4, v_5	v_2, v_1, v_5	**5**
$[S_6, D_6]$	$[v_3, v_1]$	v_3, v_4, v_5, v_1	v_3, v_2, v_1	**3**
$[S_7, D_7]$	$[v_3, v_5]$	v_3, v_4, v_5	v_3, v_2, v_1, v_5	**6**
$[S_8, D_8]$	$[v_4, v_1]$	v_4, v_5, v_1	v_4, v_2, v_1	**4**
$[S_9, D_9]$	$[v_4, v_5]$	v_4, v_5	v_4, v_2, v_1, v_5	**2**
$[S_{10}, D_{10}]$	$[v_5, v_2]$	v_5, v_1, v_2	v_5, v_4, v_2	**5**

TABLE 6
THE HIGH ORDER FINITE DIFFERENCES AND THE CONNECTION GROUPS OF EACH FIBER

Fiber,i	Optical fiber link	n(i)	m(i)	$\Delta^{m(i)}y_i$
1	$\langle v_1, v_2\rangle$	4	4	-7
2	$\langle v_2, v_1\rangle$	0	0	0
3	$\langle v_2, v_3\rangle$	1	1	3
4	$\langle v_3, v_2\rangle$	0	0	0
5	$\langle v_2, v_4\rangle$	3	3	-1
6	$\langle v_4, v_2\rangle$	0	0	0
7	$\langle v_3, v_4\rangle$	2	2	-3
8	$\langle v_4, v_3\rangle$	0	0	0
9	$\langle v_4, v_5\rangle$	5	5	18
10	$\langle v_5, v_4\rangle$	0	0	0
11	$\langle v_5, v_1\rangle$	3	2	-1
12	$\langle v_1, v_5\rangle$	0	0	0

For the linear function (5), Y_2 has a dimension of (12x1), Y_1 has a dimension of (12x1), A has a dimension of (12x10), U_{max} has a dimension of (12x1), X_n has a dimension of (10x1). Matrix A is a known matrix (12x10) that is always constant because it depends on the optical paths that are constant for this example. The unused available capacity for each optical fiber is positive or zero so there is no possible connection problem. So the linear function provides:

$$\begin{vmatrix}19\\10\\23\\21\\19\\21\\17\\27\\10\\19\\18\\17\end{vmatrix}=\begin{vmatrix}30\\30\\30\\30\\30\\30\\30\\30\\30\\30\\30\\30\end{vmatrix}-\begin{vmatrix}11\\0\\3\\0\\11\\0\\9\\0\\20\\0\\12\\0\end{vmatrix}-\begin{vmatrix}0\\20\\4\\9\\0\\9\\4\\3\\0\\11\\0\\13\end{vmatrix}$$

For the polynomial method, when no group goes through the optical fiber, then the degree of the polynomial function is 0, when one group goes through, then the degree of the polynomial function is 1, when two groups go through, then the degree of the polynomial function is 2, etc. The matrix that provides the unused capacity of all optical fibers has dimension (12x1). If the degree of polynomials increases then the writing of the polynomial numerical coefficients has error because these are difficult to be represented. This method is an accurate one but if all coefficients are not written completely with 15 digits there are errors in the results. They are rounded to the closest integer to agree with the real ones and the error values are not written. The results of the previous methods are Cavl = Cavp = Cav =221, Cwl = Cwp =Cw =66 and Cpl = Cpp =Cp =73. The network installed capacity is Cinst=12*30=360 wavelengths. So the following sum is valid

Cw+Cpr+Cav=Cinst or 66+73+221=360. The wavelength protection ratio for the sharing protection is 73/66 = 1.1 and the wavelength protection ratio for dedicated protection is 82/66 = 1.24. So the sharing protection constitutes a more attractive protection method because it has better bandwidth efficiency. The agreement of two methods (linear and polynomial) in this example is absolute with accuracy of 15 decimal digits but if there are the differences are ought to the difficulty to write all decimal digits for each polynomial coefficient of the polynomial method.

The polynomial method is written:

$$\begin{vmatrix} y_{1,5} \\ y_{2,1} \\ y_{3,2} \\ y_{4,1} \\ y_{5,4} \\ y_{6,1} \\ y_{7,3} \\ y_{8,1} \\ y_{9,6} \\ y_{10,1} \\ y_{11,4} \\ y_{12,1} \end{vmatrix} = \begin{vmatrix} 30 - 0.5*5 - 0.2777*5^2 + 0.006944*5^3 - 0.00387*5^4 \\ 10 \\ 26 - 1.5*2 \\ 21 \\ 30 - 1*4 - 0.333*4^2 - 0.026*4^3 \\ 21 \\ 26 - 1.5*3 - 0.5*3^2 \\ 27 \\ 30 - 2.5*6 - 0.055*6^2 - 0.0486*6^3 + 0.003144*6^4 + 0.00044*6^5 \\ 19 \\ 30 - 1.5*4 - 0.27*4^2 - 0.0243*4^3 \\ 17 \end{vmatrix} = \begin{vmatrix} 19 \\ 10 \\ 23 \\ 21 \\ 19 \\ 21 \\ 17 \\ 27 \\ 10 \\ 19 \\ 18 \\ 17 \end{vmatrix}$$

In the *second case*, the optimization (minimization) of the unused available network capacity is computed for this representative topology. The optimization (minimization) is generated by four ways. In the first way, the optimization is obtained by the modified algorithm (to subject to the ILP equations) setting all connection group sizes equal to 1(initial feasible solution) and computes the connection group sizes of the corresponding source-destination nodes pairs that minimize the network unused available capacity, table 7 column (1). In the second one the node pairs are sorted according to the primary path length (hops number) and the optimization is obtained as the first way, table 7, column(2). This solution obtains the best results for the network unused available capacity than setting values of 2, 3, 4 and 5. The method (1) could generate the most acceptance solutions. At first step, the algorithms (1) and (2) finds an integer optimal solution for connection group size and terminate when all node pairs have been processed. So a connection group size for each node pair and a minimum unused available network capacity are generated. At the second step either the previous results are verified or a best approach for the unused available network capacity is obtained and then terminated. In the third one, the optimization is obtained by statistical way. The unused available network capacity is computed for a large number of instances (up to 10000000) with random values of connection group size. The connection group size has adjusted, variable maximum value and the minimum unused available capacity is obtained and the algorithm terminates after exhausting all the instances, table 7 column (3). In the forth one, the optimization is obtained by a variation of the simplex algorithm. In this method, there are several scenarios of 1:1 optical path sharing protection and their number increases rapidly when the number of SD also increases. The Simplex algorithm uses one scenario each time to generate the minimum unused available capacity table 7, column (4) .SD means node pair number. It is an ILP problem and the number of the variables and the number of equations also increases rapidly with the network size so this method is impractical for large networks.

E. The execution time

The complexity of these algorithms for the node number q depends on the network size that means of the square of the node number so it is written as O (q^2). Time complexity of that

TABLE 7

THE METHOD COMPARISON

SN	SD	(1)	(2)	(3)	(4)
1	2	150	150	150	150
2	4	6	6	6	6
3	5	152	125	127	126
4	8	4	31	26	31
5	10	63	62	59	63
6	12	2	29	26	2
7	15	4	30	29	6
8	20	1	24	25	4

algorithm is ‘order q^2, O(q^2). It means the execution time depends of the network size. The total number of connections differentiates the complexity of each individual algorithm because each uses them with different way. The solution of the optimization problem is more complex. The general solution of this problem is difficult and this problem solution in general case is intractable. It is a NP problem and it means that it is not susceptible to efficient algorithmic solution. But it is transformed to NP-complete problem reducing the time in polynomial time. The finite difference methods can correctly solve the general optimization problem. The methods (1), (2) and (3) of the table 7 are generated modifying the simple algorithm and using a variation of the simplex algorithm (4) an acceptance solution is obtained. At methods (1) and (2), the starting of the solution is the initial feasible solution and search for the best solution with algorithms of two steps. The running time of (1) is about (2x21) and (2) about (2x23) hundredths of second. The running time of (2) is larger then (1) because the sorting is used. So these methods (1) and (2) are suitable for larger networks. The solution of this problem with the approach of the *random permutations* (3) is used and a large number of instances are taken so that either the optimum is obtained exactly or it is in a value range. So the random variable is the unused available network capacity and the probability distribution as well as the cumulative probability distribution could be also calculated for several numbers of instances but in this paper only the minimum value is written (table 7, column 3). This statistical method is more time consuming one. So for SD=20 and for 1000000 instances the running time is about 221 minutes without counting the time that it needs to find the maximum value, so this method is impractical for large networks when the used computer has low frequency clock (133MHz). This algorithm terminates after exhausting all the instances. It is noted that the WDM system capacity must be smaller as soon as possible for this method to reduce the consuming time. The (4) method uses several scenarios to obtain the minimum unused available capacity and their number increases rapidly when the number of SD also increases. The 1:1 optical path sharing protection and this method is used only for small networks .It is an ILP problem and the number of the variables and the number of equations also increases rapidly for larger networks so this method is impractical for them.

F. Discussion and Proposals

Today installation of WDM networks is based on mesh topologies but the latter are essentially formed by a set of point-to-point links between nodes. Network survivability is an inherent part of the mesh topology because there are usually at least two paths between end nodes. Thus, a network that uses a mesh topology can survive after a single failure.

In this protection when a cut link occurs, it is not need to know the exact location of the failure and it is capable of protecting against multiple simultaneous failures on suitable working lightpaths.

The use of the finite differences (the linear function method and the accurate polynomial function one) is possible for the study of the problems related to the protection and restoration of connections. It is shown that both methods provide arithmetical models with the same results thus they can be used successfully for verification and validation of solutions to telecommunication problems.

For a better presentation of this research a representative example is used that depicts the same results in two methods. The simple algorithm provides for each source-destination node pair and a desired connection group size, a value of the total residual available capacity of the network. The connection length depends on the number of hops. The network is a completely protectable one. Different wavelengths may be used for each connection in each hop, so that wavelength conversion is used at each node.

The optimization problem solution is more complex. The general solution of this problem is difficult so it is transformed as a NP-complete problem. The methods (1), (2) and (3) of the table 7, are generated modifying the original (simple) algorithm and using (4), an acceptance solution is obtained. It notes that ILP formulations are practical only for small networks because the number of the equations and the variables should be as less as possible. The method (4) is impractical for larger networks because there are a lot of scenarios for 1:1, it is an ILP problem and the number of the variables and the number of equations also increase rapidly. So in the example, table 7, SN=8, SD=20, it is difficult to have the best solution because I must solve one by one each scenario trying to obtain the minimum. But the algorithm (1) in about a half of the second a better solution is obtained. In the same example, table 7, SN=5, SD=10 the best solution is obtained by method (3) in a long time. The methods (1) and (2) obtain good solutions near the minimum in about a half of the second at the cost of the accuracy. The speed of the algorithms (1) and (2) is short, ought to the few (one or two) steps that they use to find a good or optimal connection group size passing through each lightpath and terminate when all node pairs have been processed obtaining a minimum unused available network capacity.

Many problems require the use of *randomly generated permutations*. The random permutation generation is done by Pascal command *random (x)* that generates random numbers in the range, 0<=num<*x*. These values are the connection group sizes and an unused available network capacity is calculated. This value is compared with the others so that the minimum is obtained.

In Turbo Pascal for the PC, the floating-point formats can be used. Such formats will force some precision to be lost in representations of the polynomial coefficients and computation results. So the methods (linear, accurate polynomial) to agree, the polynomial method must represent with accuracy of 15 decimal digits but if some differences could be appeared, they are ought to the difficulty to write all decimal digits for each polynomial coefficient of the polynomial method.

IV. CONCLUSIONS

The optical networks based on WDM technology can potentially carry large amount of data in each fiber link in the network. So either these networks could be protected suitable or its duration must be shorter as soon as possible.

This study examined the approach to survive of failures as a cut link. It is also it is capable of protecting against multiple simultaneous failures on suitable working lightpaths. The approach is based on a basic survivable paradigm of the 1:1 shared protection. In path protection schemes, backup paths and wavelengths are reserved in advance at the time connection setup. The formulated ILP is used to calculate the problem of the minimization of the unused available capacity. The above approach has been researched on the basis of the finite differences.

REFERENCES

[1] H.Levy and F.Lessman. *Finite Difference Equations.* DOVER, 1961.

[2] H. Kobayashi. *Modeling and Analysis* . ADDISON-WESLEY, 1981.

[3] Evans and Minieka. *Optimization Algorithms for Networks and Graphs* . MARCEL DEKKER, INC, 1992.

[4] T. Wu. *Fiber Network Service Survivability*. ARTECH HOUSE,1992.

[5] A.Bononi.*Optical Networking*. Part 2,SPRINGER ,1992.

[6] M. Caroll,V. J. Roese and T. Ohara. "The operator's View of OTN Evolution," *IEEE Comms Magazine* September 2010, Vol 48, No 9, pp. 46-51.

[7] M. Sridharan,V. Salapaka and A.K. Somani. "A Practical Approach to Operating Survivable WDM Networks," *IEEE JSA in Communications*, January 2002,Vol 20, No 1, pp 34-46.

[8] O. Gerstel and R. Ramaswami, Xros. "Optical Layer Survivability-A services perspective," *IEEE Comms Magazine* March 2000 ,Vol 38,No 3 , pp104-113.

[9] O. Gerstel and R. Ramaswami, Xros. "Optical Layer Survivability-An implementaion perspective*," IEEE JSA of Communication*, October 2000, Vol 18, No10, pp1885-1889.

[10] Y. Ye, S. Dixit and Mohamed Ali. "On Joint Protection /Restoration in IP CENTRIC DWDM, *"IEEE Comms Magazine*, June 2000, Vol 38, No 6, pp174-183.

[11] Y. Ye, C. Assi, S. Dixit and M. Ali. "A Simple Dynamic Integrated Scheme in Provisioning /Protection in IP over WDM Networks," *IEEE Comms Magazine*, November 2001 ,Vol 39, No 11, pp174-182.

[12] G. Carrozzo, St. Giordano ,M. Menchise and M. Pagano. "A Preplanned Local Repair Restoration Strategy for Failure Handling in Optical Transport Networks *," Photonic Network Communications* 4:3/4,345-355, 2002, Kluwer Academic Publishers.

[13] G. Conte, M.Listanti, M.Settembre and R.Sabella ."Strategy for Protection and Restoration of Optical Paths in WDM Backbone Networks for Next Generation Internet Infrastructures," *IEEE Journal of LightWave Technology*, August 2002, Vol 20, No 8, pp 1264-1276.

[14] Y. Xiong ,D. Xu and C. Qiao. "Achieving Fast and Bandwidth – Efficient Shared – Path Protection,"*IEEE Journal of LightWave Technology*, February 2003, Vol 21, No 2,pp 365-371.

[15] Canhui (Sam), J. Zhang , H. Zang , L. H. Sahasrabuddhe and B. Mukherjee. "New and Improved Approaches for Shared – Path Protection in WDM Mesh Networks," *IEEE Journal of LightWave Technology*, May 2004, Vol 22 , No 5, pp 1223-1232.

[16] S.Ramamurthy , L. Sahasrabuddhe and B. Mukherjee. "Survivable WDM Mesh Networks ," *IEEE Journal of LightWave Technology*, April 2003, Vol 21, No 4, pp 870-889.

[17] C. Sue "Wavelength Routing With Spare Reconfiguration for All-Optical WDM Networks," *IEEE Journal of LightWave Technology*, June 2005, Vol 23, No 6, pp 1991-2000.

[18] J. Burbank "Modeling and Simulation: A practical guide for network designers and developers", *IEEE Comms Magazine*, March 2009,Vol 47, No3 , pp 118.