Unified Analysis of Optimized Relay-based Wireless Systems

Ibrahim Y. Abualhaol, Senior Member, IEEE, and Mustafa M. Matalgah, Senior Member, IEEE

Abstract-In this paper, we consider the analysis of cooperative relay-based amplify-and-forward system in terms of Shannon capacity, probability of error, and outage probability over fading channels. The relay gain is optimized with the objective of maximizing the received signal-to-noiseratio (SNR) at the destination, given that the fading statistics of the links in the cooperative system are known at the relay node. The Gaussian finite mixture is utilized to mathematically formulate, in a simple and unified way, the statistics of the received SNR at optimal power gain. These statistics include the probability density function (pdf), the cumulative distribution function (CDF), the moment generating function (MGF), and the amount of fading (AoF). Using this technique, the symbol error probability (SEP) for coherent and differentially coherent modulations are derived. Analytical expressions for Shannon capacity and the outage capacity are also derived. Monte Carlo simulation results are presented to validate the derived approximate expressions.

Index Terms—Bit error rate, cooperative communication, finite mixture, spectral efficiency, frame error rate, relay-based.

I. INTRODUCTION

Cooperative relay-based communication systems exploit the broadcast nature of the wireless medium and allow nodes to jointly transmit information through relaying to improve the transmission capacity and the performance. The first formulation of a relaying problem appeared in the information theory community in [1] and were served as the motivating cause of the concurrent development of the ALOHA system at the University of Hawaii. The traditional relay channel models comprised of three nodes (Fig. 1): a source (S) that transmits information, a destination (D) that receives information, and a relay (R) that both receives and transmits information to enhance the communication between the source and the destination. This model integrated with the understanding of the benefits of MIMO (multiple-inputmultiple-output) systems in wireless channels makes the

Ibrahim Y. Abualhaol is an assistant professor at Khalifa University of Science, Technology, and Research, P.O.Box 573, Sharjah, United Arab Emirates (971-6597-8804; fax: 971-6561-1789; e-mail: ibrahimee@ieee.org).

Mustafa M. Matalgah is an associate professor at the University of Mississippi, University, MS 38677, USA (e-mail: mustafa@olemiss.edu).



Fig. 1. Relay-based cooperative communication system.

strategies designed for MIMO systems and offer significant network performance enhancements in terms of various metrics, including increased capacity, improved reliability, and minimized symbol-error probability (SEP). Therefore, the interest in distributed systems (i.e., virtual MIMO) has inspired the community to analyze the statistics of the cooperative relay-based systems over fading channels.

The authors in [2] analyzed the performance of two-hop relay-based system over Rayleigh fading channels. In this work the authors derived closed-form expressions for the pdf of the received signal-to-noise-ratio (SNR) at the destination without taking into consideration the direct path. A more general cooperative models based on parallel relays have been examined in [3] and [4]. The authors in [3] considered the outage probability analysis of a relay-based network over Nakagami-m fading channels. A closed-form outage probability expression for the case of m=1 (i.e., Rayleigh fading) was derived with the assumption of identically independent fading on the direct and relay links. The work in [3] did not include the analysis of other possible fading models nor did it provide closed-form expressions for the case of $(m \neq 1)$, which will be one of our concerns in this paper. Another important performance metric is the SEP. The derivation of exact SEP for the same cooperative relay-based network in [3] was addressed in [4]. The analysis was performed using the moment generating function (MGF) approach over Rayleigh fading channels (MPSK modulation scheme was considered). However, the authors ignored the effect of noise at the relay in their derivations, which will be another concern of our work in this paper. Another work reported in [5] focused on the derivation of closed-form expression of the bit error rate (BER) for the detect-andforward relay-based system for differential BPSK over Nakagami-m fading channels. This work involved difficult integrations that are not simplified for nonidentical fading on

Manuscript received June 26, 2011. This work is supported by Khalifa University of Science, Technology and Research, United Arab Emirates

the direct and the relay links and/or different fading models, which is also a concern in this paper.

Adaptive power allocation for cooperative relaying system with ergodic capacity analysis was presented in [6]. The power was allocated to maximize the ergodic capacity of the cooperative system. The ergodic capacity expressions involved complicated integrals for the case of Rayleigh fading. In our work, instead of allocating the power between the source and the relay, as in [6], and assuming the relay is equipped with a power gain, we optimized the relay gain taking into consideration the effect of relay noise and generalized the performance analysis for various fading channels and for different modulation techniques.

In this paper, we consider three nodes relay-based cooperative system as given in Fig. 1 where the source node is communicating with the destination node directly and indirectly through the relay node. In this model, the relay gain is optimized, following similar formulation as given in [7], to maximize the received SNR at the destination, taking into consideration the effect of relay noise. Then, the statistics of the received SNR in relay-based nonregenerative cooperative system are formulated over non identical fading channels including Rayleigh, Weibull, and Nakagami-m. After that, the outage probability, that gives information about the reliability of the system, is derived. Moreover, the average SEP is investigated for coherent and differentially coherent modulations. The ergodic capacity analysis, which was partially presented in [8], is also included in this work. The problem formulation and the analysis of the model in this paper resulted in complicated expressions that involve integrals that cannot be solved in closed-form. Therefore, we revert to an approximation technique to arrive at approximate closed-form expressions for the targeted performance metrics. The approximation is based on the finite mixture decomposition that is done using the well-known expectation maximization algorithm [9] and [10].

The remainder of this paper is organized as follows. System and channel models are introduced in section II. Formulations of the SNR statistics in the proposed model are presented in section III. In section IV, the average SEP is derived using MGF approach for coherent and differentially coherent modulations. The ergodic capacity analysis is given in section V. Numerical results are presented and validated by simulations in section VI. Finally, the paper is concluded in section VII.

II. SYSTEM AND CHANNEL MODELS

Consider a relay-based wireless communication system as given in Fig. 1, where S is the source communicating with the destination D directly and indirectly through node R, which acts as relay. Assume that node S is transmitting a signal s(t), which has normalized average power (i.e., $E[s^2(t)] = 1$), then the received complex baseband signal at R can be written as

$$r_R(t) = b\,s(t) + n_b(t),\tag{1}$$

where \tilde{b} is the fading amplitude of the channel between nodes S and R (here we assume that the fading phase can be compensated) and $n_b(t)$ is an additive white Gaussian noise (AWGN) signal with one-sided power spectral density (PSD) given as N_b . At node R, the received signal is multiplied by the gain **G** at the relay which then retransmits the signal using a different frequency band to node D. The received complex baseband signal at node D from node R can be written as

$$r_{D,1}(t) = \mathbf{G}\widetilde{c}\left(b\,s(t) + n_b(t)\right) + n_c(t),\tag{2}$$

where \tilde{c} is the fading amplitude of the channel between nodes R and D, and $n_c(t)$ is an AWGN signal with one-sided PSD N_c . On the other hand, the received complex baseband signal at node D from node S can be written as

$$r_{D,2}(t) = \tilde{a}s(t) + n_a(t), \tag{3}$$

where \tilde{a} is the fading amplitude of the channel between nodes S and D, and $n_a(t)$ is an AWGN signal with one-sided PSD N_a . Using equal gain combining (i.e., the diversity branches are co-phased) of the received signals at D from S and R (here the separation can be achieved using frequency diversity) enables us to write the overall received signal as

$$r_D(t) = (\mathbf{G}\widetilde{c}\widetilde{b} + \widetilde{a})s(t) + \mathbf{G}\widetilde{c}n_b(t) + n_c(t) + n_a(t).$$
(4)

The overall SNR at node D (i.e., $\tilde{\gamma}$) assuming $N_a = N_b = N_c = N_o$ can be written as

$$\tilde{\gamma} = \frac{E[s^2(t)]}{N_o} \frac{[\mathbf{G}\tilde{c}\tilde{b} + \tilde{a}]^2}{[(\mathbf{G}\tilde{c})^2 + 2]},$$
(5)

where $E[s^2(t)]/N_o$ is the average SNR of an AWGN channel. The choice of the relay gain **G** determines the resultant $\tilde{\gamma}$. Theoretically, we can optimize the gain **G** to maximize $\tilde{\gamma}$ as follows:

$$\frac{\partial \tilde{\gamma}}{\partial \mathbf{G}} = 0 \Longrightarrow \mathbf{G} = 2 \frac{\tilde{b}}{\tilde{a} \tilde{c}}.$$
(6)

It follows after direct substitution of ((6)) in ((5)) that the maximum $\tilde{\gamma}$ is

$$\widetilde{\gamma}_{max} = \frac{E[s^2(t)]}{N_o} \left[\frac{1}{2}\widetilde{a}^2 + \widetilde{b}^2\right] = \frac{1}{2}\widetilde{\gamma}_a + \widetilde{\gamma}_b, \tag{7}$$

where $\tilde{\gamma}_a = \tilde{a}^2/N_o$ and $\tilde{\gamma}_b = \tilde{b}^2/N_o$ are the received SNR over S-D link and S-R link, respectively. It is clear from (7) that the maximum received SNR at node D is the combination of the 1/2 received SNR over D-S link and the received SNR over the S-R link, where the S-R link is assumed to be the good link with better statistics. The pdf of γ_{max} , given that the links in are statistically independent, can be formulated by referring to [11] as

$$f_{\tilde{\gamma}_{max}}(\gamma) = \int_0^\infty f_{\tilde{a}^2} \left(\gamma - (\frac{2}{\gamma_o})\eta\right) f_{\tilde{b}^2} \left(\frac{\eta}{\gamma_o}\right) d\eta, \qquad (8)$$

where η is the dummy variable of integration and $\gamma_o = E[s^2(t)]/N_o$. Then, the MGF of $\tilde{\gamma}_{max}$ can be written as

$$M_{\tilde{\gamma}_{max}}(s) = M_{\tilde{\gamma}_a}\left(\frac{s}{2}\right) M_{\tilde{\gamma}_b}(s), \tag{9}$$

where the resultant MGF depends on the fading statistics over both links *a* and *b*. Three fading scenarios are considered here; Rayleigh, Nakagami-*m* and Weibull fading. If we assume identical independent fading in the two links (i.e., *a* and *b*), then the MGF of γ_{max} for Rayleigh, Nakagami-*m*, and Weibull can be given, respectively, as [12, Table 2.2, pp. 21]

$$M_{\tilde{\gamma}_{max}}(s) = \left(1 - \frac{s\gamma_o}{2}\right)^{-1} \left(1 - s\gamma_o\right)^{-1},\tag{10}$$

$$M_{\tilde{\gamma}_{max}}(s) = \left(1 - \frac{s\gamma_o}{2m}\right)^{-m} \left(1 - \frac{s\gamma_o}{m}\right)^{-m},\tag{11}$$

$$M_{\tilde{\gamma}_{max}}(s) = \frac{\beta}{2} (2\pi)^{\frac{2-\beta}{2}} \left(\frac{\Gamma\left(1+\frac{4}{\beta}\right)}{\gamma_o} \right)^{\frac{\beta}{2}} \left(-\frac{\sqrt{2}s}{\beta} \right)^{\beta} \\ \times \left[\mathbf{G}_{1,\beta/2}^{\beta/2,1} \left[\left(\frac{\Gamma\left(1+\frac{4}{\beta}\right)}{\gamma_o} \right)^{\frac{-\beta}{4}} \right]^{\frac{\beta}{4}} \\ \times \left(\frac{-2s}{\beta} \right)^{\beta/2} \frac{1}{1,1+2/\beta,\mathbf{K},1+(\beta-2)/\beta} \right]^{2},$$
(12)

where $\mathbf{G}_{1,\beta/2}^{\beta/2,1}(.)$ is the Meijer's G-function [12], and *m* and β are the Nakagami-*m* and the Weibull fading parameters, respectively. These MGF expressions can be used to evaluate an important standard performance metrics in cooperative system operating over fading channels. One of these performance metrics is the outage probability (P_{out}) which is defined as the probability that the received SNR (i.e., $\tilde{\gamma}_{max}$) falls below a certain threshold, γ_{th} . Mathematically we can write $P_{out}(\gamma_{th})$ as

$$\int_{0}^{\gamma_{th}} \int_{0}^{\infty} f_{\tilde{a}^{2}} \left(\gamma - (\frac{2}{\gamma_{o}}) \eta \right) f_{\tilde{b}^{2}} \left(\frac{\eta}{\gamma_{o}} \right) d\eta d\gamma$$
$$= \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} \frac{M_{\tilde{\gamma}_{max}}(-s)}{s} \exp(s\gamma_{th}) ds,$$
(13)

where σ is chosen in the region of convergence of the integral in the complex S plane. The derivation of the outage probability of the previously described system is considered to be the simplest performance metric, in terms of the involved integration, compared to the SEP and Shannon capacity. Nevertheless, the outage probability, as given in (13) and (14), is difficult to evaluate in closed-form except for the case of identical Rayleigh fading, which can be reduced to closed-form as given by [13] and [14]. Moreover, this simple case, will also involve difficult integrals to evaluate the SEP for various coherent and differentially coherent modulations. On

the other hand, if the fading channel is considered to be Weibull fading, then it is clear that the evaluated integrand involves Meijer's G-function, which is not possible to evaluate without approximation.

For the previously described difficulties, we will follow another approach based on finite mixture approximation that unifies the analysis for various fading channels for the performance metrics herein. Using this approach, as shown in the next section, the fading pdfs are represented in terms of weighted sum of Gaussian pdfs with suitable parameters. These weights and parameters represent a signature of the fading. This kind of decomposition inherits two important advantages. The first advantage is the simplification of the analysis for various identical or even nonidentical fading models for the targeted performance metrics. Secondly, this approach unifies the analysis over various fading scenarios.

The benefit of the cooperative diversity is maximized by optimizing the relay gain (i.e., **G**). On the other hand, the optimal value of **G** in ((6)) is not practical gain when the values of \tilde{a} and \tilde{c} are very small. Following similar argument as given in [12], practical choices of the relay gain **G**, to limit the output power of the relay, can be given as the following two choices:

$$\mathbf{G} = 2 \frac{(b+N_o)}{(\widetilde{a}+N_o)(\widetilde{c}+N_o)},\tag{14}$$

$$\mathbf{G} = 2 \frac{(\widetilde{b} + N_o)}{(\widetilde{a} \ \widetilde{c} + N_o)}.$$
(15)

It will be verified by simulation that such choices will produce very close statistics compared to the case of using theoretical gain G as in (6).

III. SNR STATISTICS

We considered the performance analysis of the optimized relay-based system over generalized fading channels by decomposing the pdf of the fading envelops into a finite sum of weighted Gaussian pdfs. This decomposition can be accomplished using an expectation maximization algorithm as given in Appendix A. Referring to Appendix A, the pdfs of \tilde{a}^2 and \tilde{b}^2 can be expressed mathematically as the weighted sum of Gaussian pdfs using the expectation maximization algorithm as follows:

$$f_{\tilde{a}^2}(a^2) \approx \sum_{i=1}^{N_a} \frac{w_{a,i}}{\sqrt{2\pi}\sigma_{a,i}} \exp\left(-\frac{(a^2 - \mu_{a,i})^2}{2\sigma_{a,i}^2}\right), \quad (16)$$

$$f_{\tilde{b}^2}(b^2) \approx \sum_{j=1}^{N_b} \frac{w_{b,j}}{\sqrt{2\pi}\sigma_{b,j}} \exp\left(-\frac{(b^2 - \mu_{b,j})^2}{2\sigma_{b,j}^2}\right), \quad (17)$$

where w_i , μ_i , and σ_i^2 represent the weighting coefficient, mean, and the variance, respectively, of the i^{th} weighted pdf. N_a and N_b are the number of Gaussian components which are required to approximate the pdfs of \tilde{a}^2 and \tilde{b}^2 , respectively. As we will see by numerical examples, this kind of approximation coincides with the Monte Carlo simulation to an acceptable degree of accuracy (|error| < 1 %). Using random variable transformation and by assuming, the pdf of $\tilde{\gamma}_a$ and $\tilde{\gamma}_b$ can be obtained from (16) and (17) as

$$f_{\tilde{\gamma}_{a}}(\gamma_{a}) \approx \sum_{i=1}^{N_{a}} \frac{w_{a,i}}{\gamma_{o}\sqrt{2\pi}\sigma_{a,i}} \exp\left(-\frac{\left(\gamma_{a}/\gamma_{o}-\mu_{a,i}\right)^{2}}{2\sigma_{a,i}^{2}}\right),$$
(18)

$$f_{\tilde{\gamma}_b}(\gamma_b) \approx \sum_{i=1}^{N_b} \frac{w_{b,i}}{\gamma_o \sqrt{2\pi}\sigma_{b,i}} \exp\left(-\frac{(\gamma_b/\gamma_o - \mu_{b,i})^2}{2\sigma_{b,i}^2}\right).$$
(19)

Our target in the following subsections are to formulate the pdf, the commutative distribution function (CDF), the MGF, and the amount of fading (AoF) of $\tilde{\gamma}_{max}$.

A. The MGF of $\tilde{\gamma}_{max}$

The MGF of a random variable \tilde{x} with pdf $f_{\tilde{x}}(x)$, where x > 0, is defined as

$$M_{\tilde{x}}(\mathbf{s}) = E[\exp(\mathbf{s}x)] = \int_0^\infty \exp(\mathbf{s}x) f_{\tilde{x}}(x) dx.$$
(20)

From (18) and (19), and by using (20), it can be shown after some straightforward manipulation that

$$M_{\tilde{\gamma}_{a}}(\mathbf{s}) = \sum_{i=1}^{N_{a}} w_{a,i} \exp\left[(\gamma_{o} \mu_{a,i})\mathbf{s} + \frac{(\gamma_{o}^{2} \sigma_{a,i}^{2})\mathbf{s}^{2}}{2}\right], \quad (21)$$

$$M_{\tilde{\gamma}_b}(\mathbf{s}) = \sum_{i=1}^{N_b} w_{b,i} \exp\left[(\gamma_o \mu_{b,i})\mathbf{s} + \frac{(\gamma_o^2 \sigma_{b,i}^2)\mathbf{s}^2}{2}\right].$$
 (22)

From (9), and by assuming \tilde{a} and \tilde{b} to be independent random variables, which is an acceptable assumption because of the independency between the two links, the MGF of $\tilde{\gamma}_{max}$, after scaling $\tilde{\gamma}_a$ by 1/2, can be given as follows:

$$M_{\tilde{\gamma}_{max}}(\mathbf{s}) = \sum_{i=1}^{N_a} \sum_{j=1}^{N_b} w_{ij,eq} \exp\left[(\mu_{ij,eq})\mathbf{s} + \frac{(\sigma_{ij,eq}^2)\mathbf{s}^2}{2}\right],$$
(23)

where,

$$\begin{split} w_{ij,eq} &= w_{i,a} w_{j,b} \\ \mu_{ij,eq} &= \gamma_o \Biggl[\frac{\mu_{a,i}}{2} + \mu_{b,j} \Biggr] \\ \sigma_{ij,eq}^2 &= \gamma_o^2 \Biggl[(\frac{\sigma_{a,i}}{2})^2 + \sigma_{b,j}^2 \Biggr]. \end{split}$$

B. The pdf of $\tilde{\gamma}_{max}$

By noticing that (23) represents a double summation of MGF of Gaussian random variables with parameters $\mu_{ij,eq}$ and $\sigma_{ij,eq}$, it can be proved that the pdf of $\tilde{\gamma}_{max}$ is given by

$$f_{\tilde{\gamma}_{max}}(\gamma) = \sum_{i=1}^{N_a} \sum_{j=1}^{N_b} \frac{w_{ij,eq}}{\sqrt{2\pi}\sigma_{ij,eq}} \exp\left[-\frac{(\gamma - \mu_{ij,eq})^2}{2\sigma_{ij,eq}^2}\right].$$
(25)

The derived expression of $f_{\tilde{\gamma}_{max}}(\gamma)$ contains $(N_a + N_b)$ means, $(N_a + N_b)$ variances, and $(N_a + N_b)$ weighting coefficients.

C. The CDF of $\tilde{\gamma}_{max}$

The conventional definition of the CDF of a random variable \tilde{x} is given by

$$F_{\widetilde{X}}(x) = \int_{-\infty}^{x} f_{\widetilde{X}}(x) dx.$$
 (26)

For positive-valued $\tilde{\gamma}_{max}$, the CDF can be shown to be

$$F_{\widetilde{\gamma}_{max}}(\gamma_{th}) = \sum_{i=1}^{N_a} \sum_{j=1}^{N_b} w_{ij,eq} \left[1 - Q \left(\frac{\gamma_{th} - \mu_{ij,eq}}{\sigma_{ij,eq}} \right) - Q \left(\frac{\mu_{ij,eq}}{\sigma_{ij,eq}} \right) \right],$$
(27)

where Q(x) is the Q-function defined as

$$Q(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-\frac{u^{2}}{2}) du.$$
 (28)

Detailed derivation of (27) is given in Appendix B.

D. The AoF of $\tilde{\gamma}_{max}$

The AoF of a fading channel, where $\tilde{\gamma}_{max}$ is the received SNR, is defined as

$$AoF = \frac{E[\tilde{\gamma}_{max}^2]}{E[\tilde{\gamma}_{max}]^2} - 1.$$
(29)

The AoF is considered a unified measure of the severity of fading. Typically, this performance criterion is independent of the average fading power (i.e., γ_o). The derivation of AoF can be done using the MGF in (23), which results in

$$AoF = \frac{\sum_{i=1}^{N_a} \sum_{j=1}^{N_b} w_{ij,eq} \left[\sigma_{ij,eq}^2 + \mu_{ij,eq}^2 \right]}{\left[\sum_{i=1}^{N_a} \sum_{j=1}^{N_b} w_{ij,eq} \ \mu_{ij,eq} \right]^2} - 1.$$
(30)

Detailed derivation of (30) is given in Appendix C. It is clear from (30) and noticing (24) that the AoF is independent of γ_o .

(24) IV. SYMBOL ERROR PROBABILITY

The MGF approach is very useful to derive the average SEP of a communication system over fading channels. In this section, we discuss how the MGF in (23) can be used to simplify the derivation of analytical expressions of the average SEP over generalized fading channels that can be represented as finite weighted sum of Gaussian pdfs. The author in [15] summarizes general SEP expressions over AWGN channel for coherent modulations as

TABLE I: SEP FOR VARIOUS MODULATION SCHEMES

where α and ρ depend on the modulation type. In particular, the nearest neighbor approximation has the form as given in (31), and ρ is a constant that relates the minimum distance to the average symbol energy. Table I summarizes the SEP (i.e., $P_s(\gamma)$) for PSK, QAM, and FSK modulations. Moreover, as an example of differentially coherent modulation, $P_{s}(\gamma)$ of DPSK is also given in Table 1. To get an approximation of the bit error rate (BER) we use the same expression but we replace α by $\frac{\alpha}{\log_2(M)}$, where M is the

modulation order.

The average SEP (i.e., SEP) over generalized fading channels can be derived as follows:

$$\overline{SEP} = \int_{0}^{\infty} P_{s}(\gamma) f_{\tilde{\gamma}_{max}}(\gamma) d\gamma$$
$$= \frac{\alpha}{\pi} \int_{0}^{\infty} \int_{0}^{\pi/2} \exp\left(\frac{-\rho}{2\sin^{2}(\phi)}\right) f_{\tilde{\gamma}_{max}}(\gamma) d\phi d\gamma$$
$$= \frac{\alpha}{\pi} \int_{0}^{\pi/2} M_{\tilde{\gamma}_{max}}\left(\frac{-\rho}{2\sin^{2}(\phi)}\right) d\phi.$$
(32)

Substituting (23) in (32), we get

$$\overline{SEP}; \quad \frac{\alpha}{\pi} \sum_{i=1}^{N_a} \sum_{j=1}^{N_b} w_{ij,eq} \int_0^{\pi^2} \exp\left[\frac{-\mu_{ij,eq}\rho}{2\sin^2(\phi)} + \frac{\sigma_{ij,eq}^2\rho^2}{8\sin^4(\phi)}\right] d\phi.$$
(33)

For differential modulation, the average SEP of binary differential phase shift keying (DPSK) can be shown to be

$$\overline{SEP} = \frac{1}{2} M_{\tilde{\gamma}}(s)|_{s=1}; \sum_{i=1}^{N_a} \sum_{j=1}^{N_b} \frac{w_{ij,eq}}{2} \exp[(\mu_{ij,eq}) + \frac{(\sigma_{ij,eq}^2)}{2}].$$
(34)

It is obvious from (33) and (34) that using Gaussian finite mixture representation enabled us to find expressions for the average SEP in simple forms.

Modulation	$P_s(\gamma)$
BFSK	$= Q(\sqrt{\gamma})$
BPSK	$=Q(\sqrt{2\gamma})$
DPSK	$=\frac{1}{2}\exp\left(-\gamma\right)$
QPSK,4-QAM	$\simeq \overline{2Q} \left(\sqrt{\gamma} ight)$
M-PAM	$\simeq \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6\gamma}{M^2 - 1}}\right)$
M-PSK	$\simeq 2Q \left(\sqrt{2\gamma}\sin(\pi/M)\right)$
Rectangular M-QAM	$\simeq \frac{4(\sqrt{M}-1)}{\sqrt{M}}Q\left(\sqrt{\frac{3\gamma}{M-1}}\right)$
Nonrectangular M-QAM	$\simeq 4Q(\sqrt{\frac{3\gamma}{M-1}})$

V. ERGODIC CAPACITY ANALYSIS

A. Average Capacity

The Ergodic Shannon capacity of a channel defines its theoretical upper bound for the maximum rate of data transmission at an arbitrary small BER without any delay or complexity constraints. The average Shannon capacity with receiver channel side information (CSI) can be obtained from results in [15] as

$$\overline{C} = \int_0^\infty B \log_2(1+\widetilde{\gamma}) f_{\widetilde{\gamma}}(\gamma) d\gamma, \qquad (35)$$

where B is the channel bandwidth in H_z and \overline{C} is the average ergodic capacity in bps. The for(ma) in (35) represents a probabilistic average of AWGN channel Shannon capacity $(B\log_2(1+\tilde{\gamma}_{max}))$ over the distribution of $\tilde{\gamma}_{max}$. An upper bound of the average capacity can be found using Jensen's inequality as follows:

$$C = BE[\log_2(1+\widetilde{\gamma}_{max})] \le B\log_2(1+E[\widetilde{\gamma}_{max}]), \quad (36)$$

where $E[\tilde{\gamma}_{max}]$ is the average SNR at the receiver, and it is clear from (7) that

$$E[\tilde{\gamma}_{max}] = \frac{1}{2}E[\tilde{\gamma}_a] + E[\tilde{\gamma}_b] = \left(\frac{1}{2}E[\tilde{a}^2] + E[\tilde{b}^2]\right)\gamma_o.$$
(37)

Then, the upper bound of the average capacity (i.e., \overline{C}) in a relay-based system after optimizing the relay gain G can be given as

$$B \log_2 \left(1 + \left(\frac{1}{2} \sum_{i=1}^{N_a} w_{a,i} \mu_{a,i} + \sum_{j=1}^{N_b} w_{b,j} \mu_{b,j} \right) \mathcal{B}_{\sigma} \right).$$
(38)

It can be seen from (38) that the upper bound of the average capacity is given in simple analytical expression using Gaussian finite mixture representation.

B. Outage Capacity

The outage capacity is defined as the probability that the instantaneous capacity falls bellow a certain threshold (C_{th}), which can be derived as follows:

$$P_{out}(C_{th}) = Prob[C \le C_{th}]$$

= $Prob[B \log_2(1 + \tilde{\gamma}_{max}) \le C_{th}]$
= $Prob\left[\tilde{\gamma}_{max} \le 2^{C_{th}/B} - 1\right].$

Using (25) and after integration, as in (27), the outage capacity can be shown to have the following analytical expression:

$$P_{out}(C_{th}) = F_{\tilde{\gamma}_{max}} \left(2^{C_{th}/B} - 1 \right), \tag{40}$$

where $F_{\tilde{\gamma}_{max}}(\gamma)$ is the CDF of $\tilde{\gamma}_{max}$.

VI. SIMULATION AND NUMERICAL RESULTS

In this simulation, the cooperative relay-based wireless system is assumed as shown in Fig. 1. The fading amplitudes \widetilde{a} , \widetilde{b} , and \widetilde{c} are assumed to be independent but not necessarily identical random variables. Three types of fading are chosen; Rayleigh for non line-of-sight, Nakagami-m for ostructed line-of-sight, and Weibull fading for other possible fading scenarios. For more information about these types of fading, the reader can refer to [12]. The fading amplitude \tilde{a} is assumed to be a Rayleigh random variable with $E[\tilde{a}^2] = 1$, where the S-D link is assumed to be the worst link. The fading amplitude \tilde{b} is assumed to be a Weibull random variable with $E[\tilde{b}^2] = 1$ and fading parameter ($\beta = 4$) which represents a more reliable link than the S-D link. The fading amplitude \tilde{c} is assumed to be a Nakagami-m random variable with $E[\tilde{c}^2] = 1$ and fading parameter (m=4), which represents a reliable R-D link. A finite mixture with expectation maximization algorithm, as given in Appendix A, is used to approximate the pdfs of \tilde{a}^2 , \tilde{b}^2 , and \tilde{c}^2 using ten Gaussian components for each (with tolerance $\varepsilon = 10^{-3}$). The choice of ten components is for demonstration of the analysis and can be increased if more accuracy is required. The parameters and the weighting coefficients estimation for $f_{\tilde{a}^2}(a^2)$, $f_{\tilde{b}^2}(b^2)$, and $f_{z^2}(c^2)$ are given in tables II, III, and IV, respectively.

A comparison between the analytical results for $F_{\tilde{\gamma}_{max}}(\gamma_{th})$ in (27) and the Monte Carlo simulation (10⁵ simulation runs) for $\gamma_o = 10$, 20, and 30 dB are given in Fig. 1. The analytical expression results of $F_{\tilde{\gamma}_{max}}(\gamma_{th})$ matches the results from simulation. In Fig. 2, the system performance is improved by increasing γ_o (i.e., the SNR of AWGN channel), which is a well-known fact.

TABLE II: PARAMETERS AND WEIGHTING COEFFICIENTS FOR $f_{\sim 2}(a^2)$

		u		
i	$w_{a,i}$	$\mu_{a,i}$	$\sigma^2_{a,i}$	
1	0.1139	0.2821	0.0398	
2	0.0087	4.9530	0.4716	1
3	0.6147	0.9306	0.4448	
4	0.0437	2.8815	0.2984	$\left \right\rangle$
5	0.1752	0.5280	0.1466	ĺ
6	0.0115	4.0052	0.2828	
7	0.0017	6.9460	1.3330	
8	0.0198	2.4941	0.3010	
9	0.0034	0.2566	0.0415	
10	0.0074	3.4896	0.3592	

TABLE III: PARAMETERS AND WEIGHTING COEFFICIENTS FOR $f_{-2}(b^2)$

		⁵ b ²	
i	$w_{b,i}$	$\mu_{b,i}$	$\sigma_{b,i}^2$
1	0.0920	0.6081	0.0375
2	0.7100	0.9287	0.0982
3	0.0436	0.5662	0.0337
4	0.0844	1.5697	0.0500
5	0.0271	1.9191	0.0566
6	0.0176	2.1649	0.0819
7	0.0183	1.3026	0.0809
8	0.0024	0.4655	0.0264
9	0.0023	2.5025	0.0636
10	0.0022	2.9282	0.1070

TABLE IV: PARAMETERS AND WEIGHTING COEFFICIENTS FOR $f_{z2}(c^2)$

i	$w_{c,i}$	$\mu_{c,i}$	$\sigma^2_{c,i}$
1	0.1863	1.3307	0.1051
2	0.1230	0.4337	0.0402
3	0.4457	0.8707	0.0995
4	0.1815	0.5790	0.0645
5	0.0172	0.3380	0.0267
6	0.0269	1.6683	0.0854
7	0.0104	1.9818	0.0584
8	0.0065	2.1361	0.0614
9	0.0015	2.6075	0.0672
10	0.0010	0.2962	0.0294



Fig. 2. Comparison between the analytical results for $F_{\tilde{\gamma}_{max}}(\gamma_{th})$ in (27) and the Monte Carlo simulation (10⁵ simulation runs) for different values of γ_o .

The derivation of optimum value for the relay gain (**G**) to maximize the $\tilde{\gamma}_{max}$ resulted in a nonpractical value of **G** as given in (6). Practical choices of **G** are given in (14) and (15). Fig. 3 gives simulation results of $F_{\tilde{\gamma}}$ (γ_{th}) ($\gamma_o = 10$ and 10^5 simulation runs) for various choices of relay gain: **G** = 1, **G** in (6), **G** in (14), and **G** in (15), which shows how tight the optimal CDF is with the suboptimal one. The AoF was calculated using (30) and by simulation (10^5 simulation runs) to be 0.2007 and 0.2015 (0.4% error), respectively, which shows the accuracy of the finite mixture approximation.



Fig. 3. Simulation results of $F_{\tilde{\gamma}}$ (γ_{th}) for **G** defined by (6), (14), and (16), with $\gamma_o = 10$ dB.

Comparison between the analytical results for the BER of BPSK, BFSK, and DPSK, in (33), and the Monte Carlo simulation $(10^5 \text{ simulation runs})$ are given in Fig. 4. In this

figure, the analytical results are very near to the simulation for moderate values of γ_o . On the other hand, the approximation deviate for large and small values of γ_o . This deviation is expected because of the use of approximated expression of SEP in Table I. Furthermore, to investigate our approximation with higher order modulations, a comparison between the analytical results for the BER of QPSK, 8-QAM, 16-QAM, and 64-QAM, in (33), and the Monte Carlo simulation (10⁵ simulation runs) are given in Fig. 5. Here, it is also obvious that the simulation coincide with analytical results to acceptable degree of accuracy.



Fig. 4. Comparison between the analytical results of the BER for BPSK, BFSK, and DPSK , in (33), and the Monte Carlo simulation (10^5 simulation runs).



Fig. 5. Comparison between the analytical results of the BER for QPSK, 8-QAM, 16-QAM, and 64-QAM, in (33), and the Monte Carlo simulation $(10^5 \text{ simulation runs})$.

The upper bound of the average Shannon capacity C in (38) for a relay-based system is compared with the upper bound capacity of a non-relay (i.e., direct link) system in Fig. 6. The use of a relay-based system improves the capacity and makes the upper bound analytical expression very close to simulation. Fig. 7 illustrates the comparison between the analytical results for $P_{out}(C_{th})$ in (40) and the simulation for

various values of γ_o . Such results show the possibility of analyzing the performance of cooperative relay-based system over generalized fading channels by using Gaussian finite mixture approximation.



Fig. 6. Comparison between the analytical results for the upper bound of \overline{C} , in (38), with and without relay and the Monte Carlo simulation (10⁵ simulation runs).



Fig. 7. Comparison between the analytical results of $P_{out}(C_{th})$ in (40) and the Monte Carlo simulation (10⁵ simulation runs) for various choices of γ_o .

VII. CONCLUSION

In this paper, the performance analysis of optimized cooperative relay-based wireless system over generalized fading channels is considered. General analytical formulations of SNR statistics, including pdf, CDF, MGF, and AoF, are derived in closed-forms. These expressions provided a tool to study the performance and the capacity of a relay-based system over generalized fading channels. The SEP expressions for coherent and differentially coherent modulations are given. Moreover, the outage capacity and upper bound of the average capacity are derived. In the numerical demonstrations, Weibull, Nakagami-*m*, and Rayleigh fading channels are used as examples to show the effectiveness of using finite mixture decomposition in simplifying the analysis of cooperative systems.

APPENDIX

A. Expectation Maximization Algorithm

Finite mixture is a technique for estimating the pdf of a random variable using statistical samples. In finite mixture estimation, it is assumed that a given pdf $f_{\tilde{x}}(x)$ can be estimated as a weighted sum of a g number of other pdfs. The parameters of those pdfs can be estimated using n samples, where (g << n). For the univariate case, the estimated pdf of a random variable \tilde{x} is given in [10] as

$$f_{\tilde{x}}(x) \approx \sum_{i=1}^{g} w_i \Phi_i(x; \hat{\theta}_i), \tag{41}$$

where w_i represents the weighting (mixing) coefficient of the i^{th} term, and $\Phi_i(x; \hat{\theta}_i)$ denotes the pdf with parameters represented by the vector $\hat{\theta}_i$. An important constraint on this estimation is to have $w_i > 0$ and $\sum_{i=1}^{g} w_i = 1$ to satisfy the unity integral of $f_{\tilde{x}}(x)$. The problem of estimating the parameters and the weighting coefficients via different techniques has been considered extensively in literature ([10] and references therein). One famous estimation technique is the expectation maximization algorithm [9]. In order to use the expectation maximization algorithm, we must determine the number of components, g, in the finite mixture model for required accuracy and initial estimates for the parameters and the weighting coefficients. Once we have initial estimates, we update the parameters using iterative updating equations. These equations are derived from the following equality in [9]:

$$\sum_{j=1}^{n} \tau_{ij}^{(k)} \frac{\partial}{\partial z_{i}} \sum_{i=1}^{g} \log \left[w_{i} \Phi_{i}(y_{j}; \hat{\theta}_{i}) \right] = 0, \quad (42)$$

where z_i can be either w_i or $\hat{\theta}_i$ depending on whether we are estimating w_i or $\hat{\theta}_i$, and $\tau_{ij}^{(k)}$ is the k^{th} estimated posteriori probability that the sample point y_j belongs to the i^{th} weighted pdf, which can be calculated using [9] as

$$\tau_{ij}^{(k)} = \frac{w_i^{(k)} \Phi_i(y_j; \hat{\theta}_i^{(k)})}{f_{\tilde{y}}(y_j)},$$
(43)

where i = 1, 2, K, g and j = 1, 2, K, n, g is the number of weighted pdfs, and n is the total number of sample points. Throughout the paper, the weighted normal (Gaussian) pdfs are considered. Then, the estimated pdf is given as

$$f_{\tilde{y}}(y) \approx \sum_{i=1}^{g} \frac{w_i}{\sqrt{2\pi\sigma_i}} \exp\left(-\frac{(y-\mu_i)^2}{2\sigma_i^2}\right),\tag{44}$$

where μ_i and σ_i^2 represent the mean and the variance, respectively, of the *i*th weighted pdf, which are the components of the vector $\hat{\theta}_i$ in (43). Given the constraint $\sum_{i=1}^{g} w_i = 1$ and using (43), the updating equation to estimate w_i can be derived as follows:

$$\sum_{j=1}^{n} \tau_{ij}^{(k)} \frac{\partial}{\partial w_i} \left[\log \left[\frac{w_i}{\sqrt{2\pi\sigma_i}} \exp \left(-\frac{(y_j - \mu_i)^2}{2\sigma_i^2} \right) \right] \right] + \frac{\partial}{\partial w_i} \left[\lambda \left[\sum_{i=1}^{g} w_i - 1 \right] \right] = 0$$

$$\Rightarrow \sum_{j=1}^{n} \sum_{i=1}^{n} \tau_{ij}^{(k)} + \lambda \sum_{i=1}^{n} w_i = 0 \Rightarrow \sum_{j=1}^{n} 1 + \lambda = 0 \Rightarrow \lambda = -n$$
(45)

where λ is the Lagrange multiplier associated with the constraint ($\sum_{i=1}^{g} w_i = 1$). The updating equation to estimate μ_i can be derived as follows:

$$\sum_{j=1}^{n} \tau_{ij}^{(k)} \frac{\partial}{\partial \boldsymbol{\mu}_{i}} \log \left[\frac{w_{i}}{\sqrt{2\pi\sigma_{i}}} \exp \left(-\frac{(y_{j} - \boldsymbol{\mu}_{i})^{2}}{2\sigma_{i}^{2}} \right) \right] = 0$$
$$\Rightarrow \sum_{j=1}^{n} \tau_{ij}^{(k)} y_{j} = \sum_{j=1}^{n} \tau_{ij}^{(k)} \boldsymbol{\mu}_{i} \Rightarrow \boldsymbol{\mu}_{i}^{(k+1)} = \frac{\sum_{j=1}^{n} \tau_{ij}^{(k)} y_{j}}{\sum_{j=1}^{n} \tau_{ij}^{(k)}}$$
(46)

Furthermore, the updating equation to estimate σ_i^2 can be derived as follows:

$$\sum_{j=1}^{n} \tau_{ij}^{(k)} \frac{\partial}{\partial \sigma_{\mathbf{i}}^{2}} \log \left[\frac{w_{i}}{\sqrt{2\pi}\sigma_{i}} \exp \left(-\frac{(y_{j} - \mu_{i})^{2}}{2\sigma_{i}^{2}} \right) \right] = 0$$

$$\Rightarrow \sum_{j=1}^{n} \tau_{ij}^{(k)} \frac{\partial}{\partial \sigma_{\mathbf{i}}^{2}} \left[-\frac{(y_{j} - \mu_{i})^{2}}{2\sigma_{i}^{2}} - \frac{1}{2} \log(\sigma_{i}^{2}) \right] = 0$$

$$\Rightarrow \sigma_{i}^{2(k+1)} = \frac{\sum_{j=1}^{n} \tau_{ij}^{(k)} (y_{j} - \mu_{i}^{(k)})^{2}}{\sum_{j=1}^{n} \tau_{ij}^{(k)}}.$$
(47)

Estimating the pdf of a positive-valued random variable using weighted Gaussian pdfs could result in a negative part tail for the estimated pdf. Nevertheless, this tail is negligible and can be truncated with acceptable accuracy as we will show in the numerical results. Typically, the estimation convergence can be implemented by continuing the iteration until the changes in the estimates at each iteration are less than some pre-set estimation tolerance ε . It is worthwhile to mention that the accuracy of estimation depends on two factors: the number of weighted pdfs, g, and the chosen tolerance ε . In addition, the time for convergence increases with increasing g and with decreasing ε . For more information about the time of convergence and the accuracy of the expectation maximization algorithm, the reader can refer to [9].

B. Derivation of the CDF of $\tilde{\gamma}_{max}$

Starting with the definition of the CDF in (26), the CDF of $\tilde{\gamma}_{max}$ (i.e., $F_{\tilde{\gamma}_{max}}(\gamma_{th})$) can be derived as follows:

$$\sum_{i=1}^{N_{a}} \sum_{j=1}^{N_{b}} \int_{0}^{\gamma_{th}} \frac{w_{ij,eq}}{\sqrt{2\pi\sigma_{ij,eq}}} \exp\left[-\frac{(x-\mu_{ij,eq})^{2}}{2\sigma_{ij,eq}^{2}}\right] dx$$
$$= \sum_{i=1}^{N_{a}} \sum_{j=1}^{N_{b}} \left[\frac{w_{ij,eq}}{\sqrt{2\pi\sigma_{ij,eq}}} \int_{-\frac{\mu_{ij,eq}}{\sigma_{ij,eq}}}^{\gamma_{th}-\mu_{ij,eq}} \exp(-\frac{z^{2}}{2}) dz\right]$$
$$= \sum_{i=1}^{N_{a}} \sum_{j=1}^{N_{b}} w_{ij,eq} \left[1 - Q\left(\frac{\gamma_{th}-\mu_{ij,eq}}{\sigma_{ij,eq}}\right) - Q\left(\frac{\mu_{ij,eq}}{\sigma_{ij,eq}}\right)\right].$$
(48)

C. Derivation of the AoF of $\tilde{\gamma}_{max}$

Starting with (23), which defines the MGF of $\tilde{\gamma}_{max}$, the AoF can be derived as follows:

$$E[\tilde{\gamma}_{max}] = \frac{\partial M_{\tilde{\gamma}_{max}}(s)}{\partial s}|_{s=0} = \sum_{i=1}^{N_a} \sum_{j=1}^{N_b} w_{ij,eq} \ \mu_{ij,eq},$$
$$E[\tilde{\gamma}_{max}^2] = \frac{\partial^2 M_{\tilde{\gamma}_{max}}(s)}{\partial^2 s}|_{s=0} = \sum_{i=1}^{N_a} \sum_{j=1}^{N_b} w_{ij,eq} \Big[\sigma_{ij,eq}^2 + \mu_{ij,eq}^2\Big]$$

$$\Rightarrow AoF = \frac{\sum_{i=1}^{N_a} \sum_{j=1}^{N_b} w_{ij,eq} \left[\sigma_{ij,eq}^2 + \mu_{ij,eq}^2 \right]}{\left[\sum_{i=1}^{N_a} \sum_{j=1}^{N_b} w_{ij,eq} \ \mu_{ij,eq} \right]^2} - 1.$$
(49)

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Ibrahim Abualhaol received the B.Sc. and M.Sc. degrees in electrical engineering from Jordan University of Science and Technology, Irbid, Jordan, in 2000 and 2004, respectively, and the Ph.D. degree in electrical engineering from the University of Mississippi, Oxford, MS, in August 2008. He worked as Research Assistant with the Center for Wireless Communication at the University of Mississippi, MS, form Jan 2005 until

Aug 2008. From May 2008 to Sep 2009, he worked as Wireless System Engineer with Qualcomm Incorporation and then with Broadcom Corporation at San Diego, CA. He is currently an Assistant Professor at Khalifa University of Science, Technology, and Research at Sharjah, United Arab Emirates. His research interests include OFDM, space–time coding, cooperative networks, and MIMO wireless communications. He is a senior member of IEEE, a member of Phi Kappa Phi, and a member of Sigma Xi.



Mustafa M. Matalgah received his Ph.D. in Electrical and Computer Engineering in 1996 from The University of Missouri, Columbia. From 1996 to 2002, he was with Sprint, Kansas City, MO, USA, where he led various projects dealing with optical transmission systems and the evaluation and assessment of 3G wireless communication emerging technologies. In 2000, he was a Visiting Assistant Professor at The

University of Missouri, Kansas City, MO, USA. Since August 2002, he has been with The University of Mississippi in Oxford where he is now an Associate Professor in the Electrical Engineering Department. In Summer 2008, he was a Visiting Professor at Chonbuk National University in South Korea. His current technical and research experience is in the fields of emerging wireless communications systems and communication networks. He also previously published in the fields of signal processing and optical binary matched filters. He has published more than 80 refereed journal and conference proceeding papers, five book chapters, and more than 20 technical industrial applied research reports in these areas. Dr. Matalgah received several certificates of recognition for his work accomplishments in the industry and academia. He is the recipient of the Best Paper Award of the IEEE ISCC 2005, La Manga del Mar Menor, Spain. He is the recipient of the 2006 School of Engineering Junior Faculty Research Award at The University of Mississippi. He serves as a reviewer for several international journals and conferences and served on several international conferences technical program and organization committees.